

CALCULUS
WORKSHEET 2 ON LIMITS

1. Given that $\lim_{x \rightarrow a} f(x) = -3$, $\lim_{x \rightarrow a} g(x) = 0$, $\lim_{x \rightarrow a} h(x) = 8$, find the limits that exist. If the limit does not exist, explain why.

(a) $\lim_{x \rightarrow a} [f(x) + h(x)] =$

(b) $\lim_{x \rightarrow a} [f(x)]^2 =$

(c) $\lim_{x \rightarrow a} \sqrt[3]{h(x)} =$

(d) $\lim_{x \rightarrow a} \frac{1}{f(x)} =$

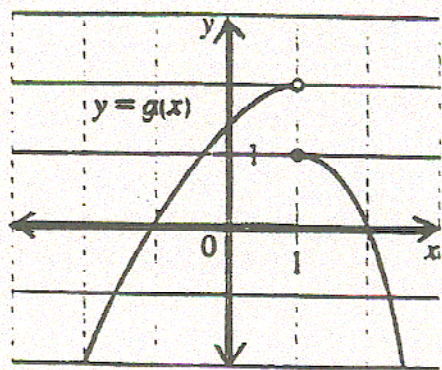
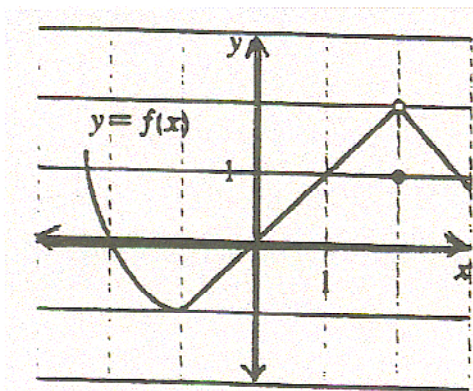
(e) $\lim_{x \rightarrow a} \frac{f(x)}{h(x)} =$

(f) $\lim_{x \rightarrow a} \frac{g(x)}{f(x)} =$

(g) $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} =$

(h) $\lim_{x \rightarrow a} \frac{2f(x)}{h(x) - f(x)} =$

2. The graphs of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.



(a) $\lim_{x \rightarrow 2} [f(x) + g(x)] =$

(b) $\lim_{x \rightarrow 1} [f(x) + g(x)] =$

(c) $\lim_{x \rightarrow 0} [f(x)g(x)] =$

(d) $\lim_{x \rightarrow -1} \frac{f(x)}{g(x)} =$

(e) $\lim_{x \rightarrow 2} x^3 f(x) =$

(f) $\lim_{x \rightarrow 1} \sqrt{3 + f(x)} =$

Find the following limits. Show all steps.

3. $\lim_{x \rightarrow 0} \frac{\sin 2x}{x} =$

$$4. \lim_{x \rightarrow 0} \frac{\sin x}{2x^2 - x} =$$

$$5. \lim_{x \rightarrow 0} \frac{x + \sin x}{x} =$$

$$6. \lim_{x \rightarrow 0} \frac{\sin^2 x}{x} =$$

$$7. \lim_{x \rightarrow 0} \frac{3 \sin 4x}{\sin 3x} =$$

$$8. \lim_{x \rightarrow 0} \frac{x^2}{1 - \cos x} =$$

9. Graph $y = x$, $y = -x$, and $y = x \cos\left(\frac{50\pi}{x}\right)$ on the same graph over the x -interval from -1 to 1 , and use the Squeeze Theorem to find $\lim_{x \rightarrow 0} x \cos\left(\frac{50\pi}{x}\right)$.

10. Sketch the graphs of $y = 1 - x^2$, $y = \cos x$, and $y = f(x)$, where f is any continuous function that satisfies the inequality $1 - x^2 \leq f(x) \leq \cos x$ for all x in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. What can you say about the limit of $f(x)$ as $x \rightarrow 0$? Explain your reasoning.

11. If $1 \leq f(x) \leq x^2 + 2x + 2$ for all x , find $\lim_{x \rightarrow -1} f(x)$.

12. If $3x \leq f(x) \leq x^3 + 2$, evaluate $\lim_{x \rightarrow 1} f(x)$.