Worksheet #1 1st FTC

① $y' = 2 + \frac{1}{x^2}$ and $y(1) = 6$. Find $y(3)$.

Method 1 $\quad y = \int \left(2 + \frac{1}{x^2}\right) dx = 2x - \frac{1}{x} + C$

$$y(1) = 6 = 2 - 1 + C$$
$$5 = C$$
$$y = 2x - \frac{1}{x} + 5$$
$$y(3) = 6 - \frac{1}{3} + 5 = \boxed{10\tfrac{2}{3} \text{ or } \tfrac{32}{3}}$$

Method 2 $\quad \int_1^3 y'\,dx = y(3) - y(1)$

$$y(3) = y(1) + \int_1^3 y'\,dx$$
$$y(3) = 6 + \int_1^3 \left(2 + \frac{1}{x^2}\right) dx$$
$$= 6 + \left[2x - \frac{1}{x}\right]_1^3$$
$$= 6 + \left(6 - \frac{1}{3}\right) - (2 - 1) = \boxed{10\tfrac{2}{3} \text{ or } \tfrac{32}{3}}$$

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② $f'(x) = \cos(2x)$ and $f(0) = 3$. Find $f\left(\frac{\pi}{4}\right)$.

Method 1 $\quad f(x) = \int \cos(2x)\,dx = \frac{1}{2}\int \cos u\,du = \frac{1}{2}\sin u + C$

$$= \frac{1}{2}\sin(2x) + C$$

$u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$$f(0) = 3 = 0 + C$$
$$3 = C$$
$$f(x) = \frac{1}{2}\sin(2x) + 3$$
$$f\left(\frac{\pi}{4}\right) = \frac{1}{2} + 3 = \boxed{3\tfrac{1}{2} \text{ or } \tfrac{7}{2}}$$

Method 2 $\quad \int_0^{\pi/4} f'(x)\,dx = f\left(\frac{\pi}{4}\right) - f(0)$

$$f\left(\frac{\pi}{4}\right) = f(0) + \int_0^{\pi/4} f'(x)\,dx$$
$$f\left(\frac{\pi}{4}\right) = 3 + \int_0^{\pi/4} \cos(2x)\,dx$$

$u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$$= 3 + \frac{1}{2}\int_0^{\pi/2} \cos u\,du = 3 + \frac{1}{2}\left[\sin u\right]_0^{\pi/2}$$
$$= 3 + \frac{1}{2}(1 - 0) = \boxed{3\tfrac{1}{2} \text{ or } \tfrac{7}{2}}$$

③ $\frac{dw}{dt} = \frac{1}{75}(600 + 20t - t^2)$ $\quad w(0) = 150$

Find $w(24)$

Method 1 $\quad W = \frac{1}{75}\int (600 + 20t - t^2)\,dt$

$$W = \frac{1}{75}\left(600t + 10t^2 - \frac{t^3}{3}\right) + C$$

$$w(0) = 150 = C$$

$$W = \frac{1}{75}\left(600t + 10t^2 - \frac{t^3}{3}\right) + 150$$

$$w(24) = \boxed{357.36 \text{ gallons}}$$

Method 2 $\quad \int_0^{24} w'(t)\,dt = w(24) - w(0)$

$$w(24) = w(0) + \int_0^{24} w'(t)\,dt$$

$$w(24) = 150 + \int_0^{24} \frac{1}{75}(600 + 20t - t^2)\,dt$$

$$= 150 + \frac{1}{75}\left[600t + 10t^2 - \frac{t^3}{3}\right]_0^{24}$$

$$= \boxed{357.36 \text{ gallons}}$$

or $\quad 150 + \frac{1}{75}\left(600(24) + 10(24)^2 - \frac{24^3}{3}\right) - \frac{1}{75}(0)$

(4) $f'(x) = \cos(x^3)$, $f(\underline{0}) = 2$.

$$f(\underline{1}) = f(0) + \int_0^1 f'(x)\,dx$$

$$\Rightarrow f(\underline{1}) = 2 + \int_0^1 \cos(x^3)\,dx = \boxed{2.932}$$

(5) $f'(x) = e^{-x^2}$, $f(\underline{5}) = 1$

$\int_{\text{lower (smaller)}}^{\text{upper (larger)}}$

$$f(2) = f(5) - \int_2^5 f'(x)\,dx$$

$$\Rightarrow f(\underline{2}) = 1 - \int_2^5 e^{-x^2}\,dx = \boxed{0.996}$$

$$\int_2^5 = -\int_5^2$$

(6) $v(t) = 5\sin(t^2)$. At $t = 6$, position is $(4, 0)$.

$$x(7) = x(6) + \int_6^7 x'(t)\,dt$$

$$\Rightarrow x(7) = 4 + \int_6^7 5\sin(t^2)\,dt = \boxed{3.837}$$

so $\boxed{\text{position} = (3.837, 0)}$

(7) $F'(t) = 2^t$ million bacteria/hr. $F(0) = 4$ million

$$F(3) = F(0) + \int_0^3 F'(t)\,dt$$

$$\Rightarrow F(3) = 4 + \int_0^3 2^t\,dt = \boxed{14.099 \text{ million}}$$

$\boxed{\text{Total increase} = 10.099 \text{ million}}$

(8) $v(t) = \dfrac{t}{1+t^2}$, $s(0) = 5$

$$s(3) = s(0) + \int_0^3 s'(t)\,dt$$

$$\Rightarrow s(3) = 5 + \int_0^3 \frac{t}{1+t^2}\,dt = \boxed{6.151}$$

CALCULUS
WORKSHEET ON THE FUNDAMENTAL THEOREM OF CALCULUS

Work the following on **notebook paper**.
Work problems 1 - 3 by both methods.

1. $y' = 2 + \frac{1}{x^2}$ and $y(1) = 6$. Find $y(3)$.

2. $f'(x) = \cos(2x)$ and $f(0) = 3$. Find $f\left(\frac{\pi}{4}\right)$.

3. Water flows into a tank at a rate of $\frac{dW}{dt} = \frac{1}{75}\left(600 + 20t - t^2\right)$, where $\frac{dW}{dt}$ is measured in gallons per hour and t is measured in hours. If there are 150 gallons of water in the tank at time $t = 0$, how many gallons of water are in the tank when $t = 24$?

Work problems 4 – 8 using the Fundamental Theorem of Calculus and your calculator.

4. $f'(x) = \cos\left(x^3\right)$ and $f(0) = 2$. Find $f(1)$.

5. $f'(x) = e^{-x^2}$ and $f(5) = 1$. Find $f(2)$.

6. A particle moving along the x-axis has position $x(t)$ at time t with the velocity of the particle $v(t) = 5\sin\left(t^2\right)$. At time $t = 6$, the particle's position is (4, 0). Find the position of the particle when $t = 7$.

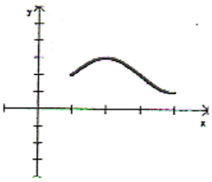
7. Let $F(t)$ represent a bacteria population which is 4 million at time $t = 0$. After t hours, the population is growing at an instantaneous rate of 2^t million bacteria per hour. Find the total increase in the bacteria population during the first three hours, and find the population at $t = 3$ hours.

8. A particle moves along a line so that at any time $t \geq 0$ its velocity is given by $v(t) = \frac{t}{1+t^2}$. At time $t = 0$, the position of the particle is $s(0) = 5$. Determine the position of the particle at $t = 3$.

Use the Fundamental Theorem of Calculus and the given graph.

9. The graph of f' is shown on the right.

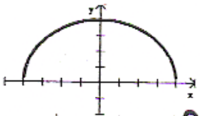
$\int_1^4 f'(x)\,dx = 6.2$ and $f(1) = 3$. Find $f(4)$.



$f(4) = f(1) + \int_1^4 f'(x)\,dx$

$f(4) = 3 + 6.2 = \boxed{9.2}$

10. The graph of f' is the semicircle shown on the right.
Find $f(-4)$ given that $f(4) = 7$.

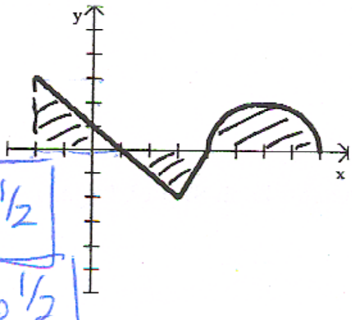


$f(-4) = f(4) - \int_{-4}^{4} f'(x)\,dx$

$f(-4) = \boxed{7 - 8\pi}$

11. The graph of f', consisting of two line segments and a semicircle, is shown on the right. Given that $f(-2) = 5$, find:

(a) $f(1)$ (b) $f(4)$ (c) $f(8)$



(a) $f(1) = 5 + \frac{1}{2}(3)(3) = \boxed{9\frac{1}{2}}$ $9\frac{1}{2}$

(b) $f(4) = 9\frac{1}{2} - \frac{1}{2}(3)(2) = \boxed{6\frac{1}{2}}$ $6\frac{1}{2}$

(c) $f(8) = \boxed{6\frac{1}{2} + 2\pi}$ $6\frac{1}{2} + 2\pi$

12. Let f be the function whose graph goes through the point (3, 6) and whose derivative is given by $f'(x)=\frac{1+e^x}{-x^2}$ Find $f(3.1)$

$\int_3^{3.1} f'(x)\,dx = f(3.1) - f(3)$

$f(3.1) = 6 + \int_3^{3.1} \frac{1+e^x}{x^2}\,dx = \boxed{6.238}$

5.762 if minus

6 + .238

13. (Multiple Choice) If f is the antiderivative of $\frac{x^2}{1+x^5}$ such that $f(1)=5$, then $f(4)=$

(A) 4.988 (B) 5 (C) 5.016 (D) 5.376 (E) 5.629

$\frac{x^2}{1+x^5}$ is deriv. of

$\int_1^4 f'(x)\,dx = f(4) - f(1)$

$f(4) = 5 + \int_1^4 \frac{x^2}{1+x^5}\,dx = \boxed{5.376}$

so $\frac{x^2}{1+x^5} = f'(x)$

Answers to Worksheet on the First Fundamental Theorem of Calculus

1. $\frac{32}{3}$
2. $\frac{7}{2}$
3. 357.36 gallons
4. 2.932
5. 0.996
6. 3.837
7. 10.099 million, 14.099 million
8. 6.151
9. 9.2
10. $7-8\pi$
11. (a) 9.5 (b) 6.5 (c) $6.5+2\pi$
12. 6.238
13. D

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