

CALCULUS
 WORKSHEET ON SECOND FUNDAMENTAL THEOREM
 AND FUNCTIONS DEFINED BY INTEGRALS

1. Find the derivatives of the functions defined by the following integrals:

(a) $\int_0^x \frac{\sin t}{t} dt$

(b) $\int_0^x e^{-t^2} dt$

(c) $\int_1^{\cos x} \frac{1}{t} dt$

(d) $\int_0^1 e^{\tan^2 t} dt$

(e) $\int_x^{x^2} \frac{1}{2t} dt, x > 0$

(f) $\int_x^2 \cos(t^2) dt$

(g) $\int_1^{\sqrt{x}} \frac{s^2}{s^2 + 1} ds$

(h) $\int_{-5}^{\cos x} t \cos(t^3) dt$

(i) $\int_{\tan x}^{17} \sin(t^4) dt$

2. The graph of a function f consists of a semicircle and two line segments as shown. Let g be the function given by

$$g(x) = \int_0^x f(t) dt.$$

(a) Find $g(0), g(3), g(-2),$ and $g(5).$

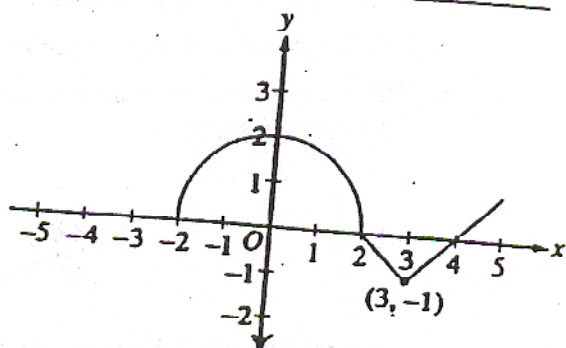
(b) Find all values of x on the open interval $(-2, 5)$ at which g has a relative maximum. Justify your answers.

(c) Find the absolute minimum value of g on the closed interval $[-2, 5]$ and the value of x at which it occurs. Justify your answer.

(d) Write an equation for the line tangent to the graph of g at $x = 3.$

(e) Find the x -coordinate of each point of inflection of the graph of g on the open interval $(-2, 5).$ Justify your answer.

(f) Find the range of $g.$



3. Let $g(x) = \int_0^x f(t) dt$, where f is the function whose graph is shown.

(a) Evaluate $g(0), g(1), g(2),$ and $g(6).$

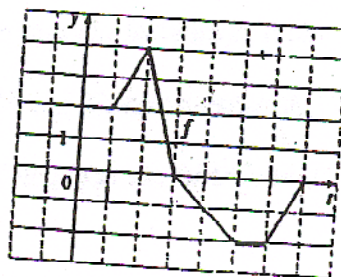
(b) On what intervals is g increasing?

(c) Where does g have a maximum value? What is the maximum value?

(d) Where does g have a minimum value? What is the minimum value?

(e) Sketch a rough graph of g on $[0, 7].$

(f) Sketch a graph of g'' on $(0, 7).$



4. Let $g(x) = \int_{-3}^x f(t) dt$, where f is the function whose graph is shown.

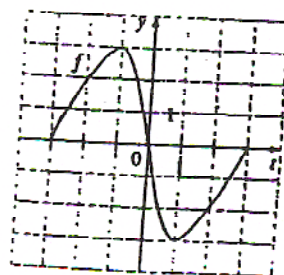
(a) Evaluate $g(-3)$ and $g(3).$

(b) At what values of x is g increasing? Justify.

(c) At what values of x does g have a maximum value? Justify.

(d) At what values of x does g have a minimum value? Justify.

(e) At what values of x does g have an inflection point? Justify.



5. Use the function f in the figure and the function g defined by

$$g(x) = \int_0^x f(t) dt.$$

(a) Complete the table.

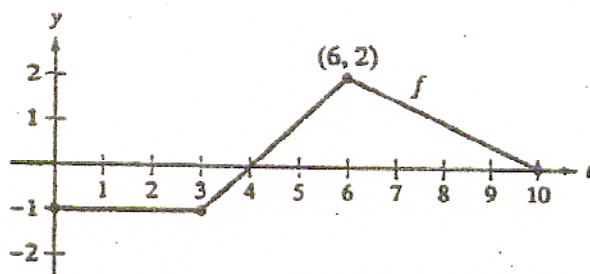
x	0	1	2	3	4	5	6	7	8	9	10
$g(x)$											

(b) Plot the points from the table in part (a).

(c) Where does g have its minimum?

(d) Which four consecutive points are collinear?

(e) Between which two consecutive points does g increase at the greatest rate?



6. If $F(x) = \int_0^x f(t) dt$

a. Identify all critical numbers of $F(x)$.

b. On what interval(s) is $F(x)$ decreasing?

c. On what interval(s) is $F(x)$ concave up?

