Worksheet

① (a) $\frac{d}{dx}\int_0^x \frac{\sin t}{t}\,dt = \boxed{\frac{\sin x}{x}}$

(b) $\frac{d}{dx}\int_0^x e^{-t^2}\,dt = \boxed{e^{-x^2}}$

★ (c) $\frac{d}{dx}\int_1^{\cos x} \frac{1}{t}\,dt = \boxed{\left(\frac{1}{\cos x}\right)(-\sin x)} = \boxed{-\tan x}$

Tricky ★ (d) $\frac{d}{dx}\underbrace{\int_0^1 e^{\tan^2 t}\,dt}_{\text{constant}} = \boxed{0}$ (derivative of a constant)

★★ (e) $\frac{d}{dx}\int_x^{x^2} \frac{1}{2t}\,dt = \frac{d}{dx}\int_x^a \frac{1}{2t}\,dt + \frac{d}{dx}\int_a^{x^2} \frac{1}{2t}\,dt$

Split

$$= -\frac{1}{2x} + \left(\frac{1}{2\cancel{x^2}_{\,x}}\right)(2\cancel{x}) = -\frac{1}{2x} + \frac{2}{2x} = \boxed{\frac{1}{2x}}$$

(f) $\frac{d}{dx}\int_x^2 \cos(t^2)\,dt = \boxed{-\cos(x^2)}$

★ (g) $\frac{d}{dx}\int_1^{\sqrt{x}} \frac{s^2}{s^2+1}\,ds = \left(\frac{x}{x+1}\right)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) = \boxed{\frac{x}{2\sqrt{x}\,(x+1)}}$

★ (h) $\frac{d}{dx}\int_{-5}^{\cos x} t\cos(t^3)\,dt =$

$$= \boxed{(\cos x)(\cos(\cos^3 x))(-\sin x)}$$

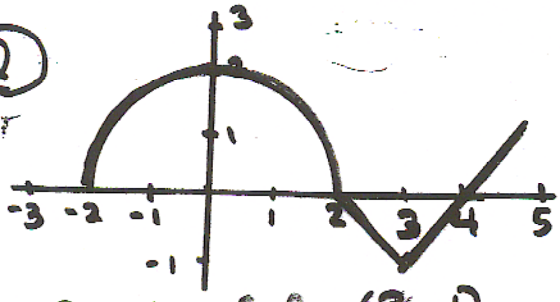
$$= \boxed{-\sin x \cos x \cos(\cos^3 x)}$$

★ (i) $\frac{d}{dx}\int_{\tan x}^{17} \sin(t^4)\,dt = \boxed{-\sin(\tan^4 x)\sec^2 x}$

← backwards so reverse limits & put negative

$$= -\frac{d}{dx}\int_{17}^{\tan x} \sin(t^4)\,dt$$

2



Graph of f
= Graph of g'

$\underline{g(x) = \int_0^x f(t)\,dt}$

$g'(x) = f(x)$

(a) $g(0) = \int_0^0 f(t)\,dt = \boxed{0}$

$g(3) = \int_0^3 f(t)\,dt$

$= \frac{1}{4}(4\pi) - \frac{1}{2}(1)(1)$

$= \boxed{\pi - \frac{1}{2}}$

$g(-2) = \int_0^{-2} f(t)\,dt = -\int_{-2}^{0} f(t)\,dt$

$= -\frac{1}{4}(4\pi) = \boxed{-\pi}$

$g(5) = \int_0^5 f(t)\,dt$

$= \pi - \frac{1}{2}(2)(1) + \frac{1}{2}(1)(1)$

$= \boxed{\pi - \frac{1}{2}}$

(b) g has a relative maximum at $\underline{x = 2}$ because $g'(x) = f(x)$ changes from positive to negative at $x = 2$.

(c)

x	$g(x)$
-2	$-\pi$
2	π
4	$\pi - 1$
5	$\pi - \frac{1}{2}$

$g(2) = \int_0^2 f(t)\,dt = \frac{1}{4}(4\pi) = \pi$

$g(4) = \int_0^4 f(t)\,dt = \pi - \frac{1}{2}(2)(1) = \pi - 1$

The absolute minimum value of g is $\underline{-\pi}$, which occurs at $x = -2$.

(d) $g'(x) = f(x)$ so $g'(3) = f(3) = -1 =$ slope

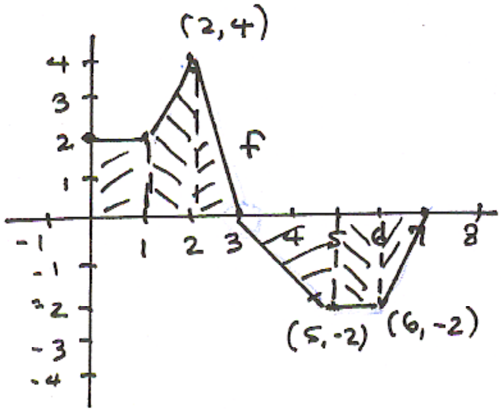
$g(3) = \pi - \frac{1}{2}$ from (a) so $(3, \pi - \frac{1}{2}) =$ point

$\boxed{y - (\pi - \frac{1}{2}) = -1(x - 3)}$

(e) g has an inflection point at $x = 0$ and at $x = 3$ because $g'(x) = f(x)$ changes from increasing to decreasing or vice versa.

(f) Range is $\boxed{[-\pi, \pi]}$.

③



$g(x) = \int_0^x f(t)\,dt$

(a) $g(0) = \int_0^0 f(t)\,dt = \boxed{0}$

$g(1) = \int_0^1 f(t)\,dt = \boxed{2}$

$g(2) = \int_0^2 f(t)\,dt =$

$= 2 + \frac{1}{2}(1)(2+4) = \boxed{5}$

$g(6) = \int_0^6 f(t)\,dt$

$= 5 + \frac{1}{2}(1)(4) - \frac{1}{2}(2)(2) - 2$

$= \boxed{3}$

(b) g is increasing on (0, 3) because $g'(x) = f(x)$ is positive on (0, 3).

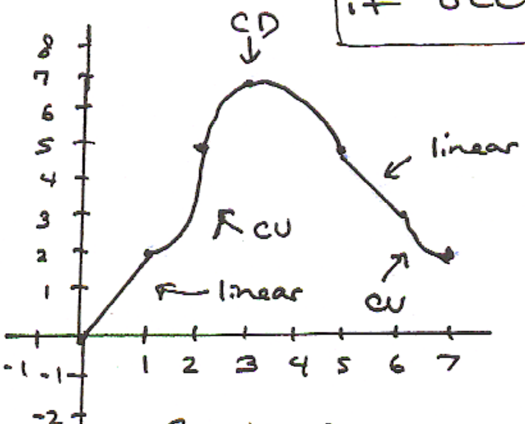
(c) g has a local maximum at $x = 3$ because $g'(x) = f(x)$ is changing from positive to negative at $x = 3$. The maximum value is 7.

(d)

x	g(x)
0	0
3	7
7	2

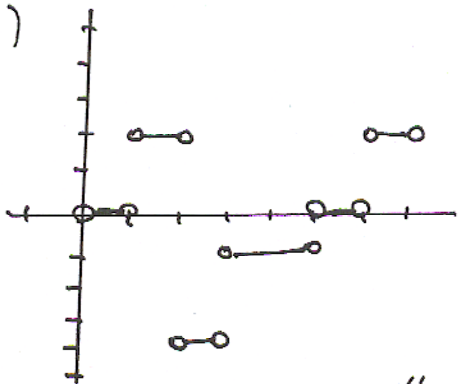
g does not have a relative minimum because $g'(x) = f(x)$ does not change from neg. to pos. so the abs. min. must lie at an endpoint. $g(0) = 0$ is the abs. min. value of g, and it occurs at $x = 0$.

(e)



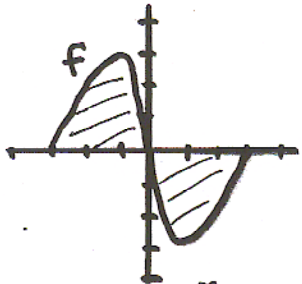
Graph of g

(f)



Graph of g''

④



$g(x) = \int_{-3}^{x} f(t)\,dt$

(a) $g(-3) = \int_{-3}^{-3} f(t)\,dt = 0$

$g(3) = \int_{-3}^{3} f(t)\,dt = 0$

(b) $g'(x) = \frac{d}{dx}\int_{-3}^{x} f(t)\,dt$

★ $\boxed{g'(x) = f(x)}$

$g'(x)$		$+$		$-$	
$g(x)$	-3	$\nearrow$	0	$\searrow$	3

$g'(x)$ is positive on $(-3, 0)$ so g is increasing on $(-3, 0)$.

(c) and (d) Rel. max. at $x = 0$, no rel. min.

so candidates are $x = -3$, $x = 0$, $x = 3$

$g(-3) = 0$

$g(0) = \int_{-3}^{0} f(t)\,dt$ $\quad g(0) =$ positive

$g(3) = 0$

x	$g(x)$
-3	0
0	positive
3	0

(c) Maximum at $x = 0$

(d) Minimum at $x = -3$ and at $x = 3$

(e) g' changes from increasing to decreasing at $x = -1$ so g has an inflection point at $x = -1$.

g' changes from decreasing to increasing at $x = 1$ so g has an inflection point at $x = 1$.

5. Use the function f in the figure and the function g defined by

$$g(x) = \int_0^x f(t)\,dt. \quad \text{so } g'(x) = f(x)$$

On $(0, 3)$, g is linear
On $(3, 6)$, g is C.U.
On $(6, 10)$, g is C.D.

(a) Complete the table.

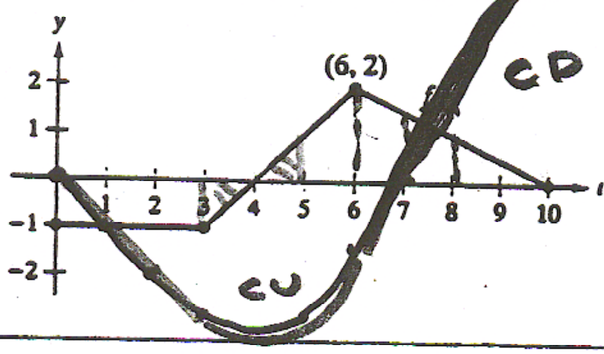
x	0	1	2	3	4	5	6	7	8	9	10
$g(x)$	0	-1	-2	-3	-3.5	-3	$-\frac{3}{2}$	$\frac{1}{4}$	$\frac{3}{2}$	$\frac{9}{4}$	$\frac{5}{2}$

(b) Plot the points from the table in part (a).

(c) Where does g have its minimum? At $x = 4$

(d) Which four consecutive points ~~are~~ on g collinear? $(0,0), (1,-1), (2,-2), (3,-3)$

(e) Between which two consecutive points does g increase at the greatest rate? Betw. $x = 6$ + $x = 7$



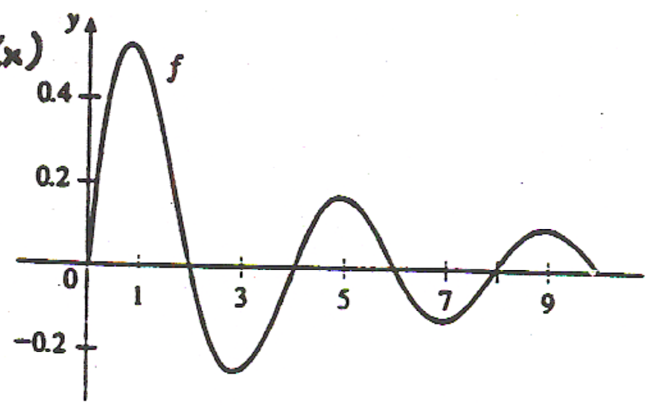
Slope from $x = 6$ to $x = 7$

$$= \frac{\frac{1}{4} - -\frac{3}{2}}{7-6} = \frac{7}{4}$$

Slope from $x = 7$ to $x = 8$

$$= \frac{\frac{3}{2} - \frac{1}{4}}{8-7} = \frac{5}{4}$$

6. If $F(x) = \int_0^x f(t)\,dt$ so $F'(x) = f(x)$



a. Identify all critical numbers of F(x).
$x = 2, 4, 6, 8$

b. On what interval(s) is F(x) decreasing?

c. On what interval(s) is F(x) concave up?

b. F is decreasing where $F'(x) = f(x)$ is negative
so $(2, 4)$ and $(6, 8)$

c. F is concave up where $F'(x) = f(x)$ is increasing
so $(0, 1)$, $(3, 5)$, and $(7, 9)$