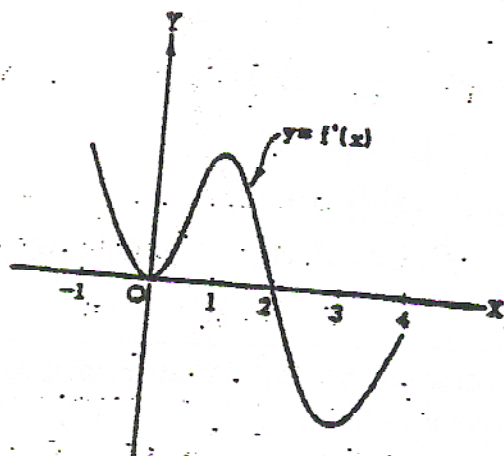


1.



Graph of  $f'$

Let  $f$  be a function that has domain the closed interval  $[-1, 4]$  and range the closed interval  $[-1, 2]$ . Let  $f(-1) = -1$ ,  $f(0) = 0$ , and  $f(4) = 1$ . Also let  $f$  has the derivative function  $f'$  that is continuous and that has the graph shown in the figure above.

- Find all values of  $x$  for which  $f$  assumes a relative maximum. Justify your answer.
- Find all values of  $x$  for which  $f$  assumes its absolute minimum. Justify your answer.
- Find the intervals on which  $f$  is concave downward.
- Give all values of  $x$  for which  $f$  has a point of inflection.
- Sketch the graph of  $f$ .

2. A function  $f$  is continuous on the closed interval  $[-3, 3]$  such that  $f(-3) = 4$  and  $f(3) = 1$ . The functions  $f'$  and  $f''$  have the properties given in the table below.

$x$	$-3 < x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 3$
$f'(x)$	Positive	Fails to exist	Negative	0	Negative
$f''(x)$	Positive	Fails to exist	Positive	0	Negative

- What are the  $x$ -coordinates of all absolute maximum and absolute minimum points of  $f$  on the interval  $[-3, 3]$ ? Justify your answer.
- What are the  $x$ -coordinates of all points of inflection of  $f$  on the interval  $[-3, 3]$ ? Justify your answer.
- Sketch a graph that satisfies the given properties of  $f$ .

5. If  $f$  is a continuous function defined for all real number  $x$  and if the maximum value of  $f(x)$  is 5 and the minimum value of  $f(x)$  is  $-7$ , then which of the following must be true?

I. The maximum value of  $f(|x|)$  is 5.

II. The maximum value of  $|f(x)|$  is 7.

III. The minimum value of  $f(|x|)$  is 0.

- (A) I only      (B) II only      (C) I and II only      (D) II and III only      (E) I, II, and III

6. An equation of the line tangent to  $y = x^3 + 3x^2 + 2$  at its point of inflection is

(A)  $y = -6x - 6$

(B)  $y = -3x + 1$

(C)  $y = 2x + 10$

(D)  $y = 3x - 1$

(E)  $y = 4x + 1$

7. If the graph of  $y = x^3 + ax^2 + bx - 4$  has a point of inflection at  $(1, -6)$ , what is the value of  $b$ ?

(A)  $-3$

(B)  $0$

(C)  $1$

(D)  $3$

(E) It cannot be determined from the information given.

8. For what value of  $x$  does the function  $f(x) = (x-2)(x-3)^2$  have a relative maximum?

(A)  $-3$

(B)  $-\frac{7}{3}$

(C)  $-\frac{5}{2}$

(D)  $\frac{7}{3}$

(E)  $\frac{5}{2}$

9. Let  $f$  be the function defined by  $f(x) = \begin{cases} x^3 & \text{for } x \leq 0 \\ x & \text{for } x > 0 \end{cases}$ . Which of the following statements

about  $f$  is true?

(A)  $f$  is an odd function.

(B)  $f$  is discontinuous at  $x = 0$ .

(C)  $f$  has a relative maximum.

(D)  $f'(0) = 0$

(E)  $f'(x) > 0$  for  $x \neq 0$

10. If  $f(x) = \sin\left(\frac{x}{2}\right)$ , then there exists a number  $c$  in the interval  $\frac{\pi}{2} < x < \frac{3\pi}{2}$  that satisfies the conclusion of the Mean Value Theorem. Which of the following could be  $c$ ?

(A)  $\frac{2\pi}{3}$

(B)  $\frac{3\pi}{4}$

(C)  $\frac{5\pi}{6}$

(D)  $\pi$

(E)  $\frac{3\pi}{2}$