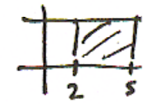
CALCULUS

WORKSHEET 2 ON FUNDAMENTAL THEOREM OF CALCULUS

1. If $f(1)=12$, f' is continuous, and $\int_1^4 f'(x)\,dx=17$, what is the value of $f(4)$?

$f(4)=f(1)+\int_1^4 f'(x)\,dx = 12+17=\boxed{29}$

$\int_2^5 3\,dx$



2. If $\int_2^5 (2f(x)+3)\,dx=17$, find $\int_2^5 f(x)\,dx$.

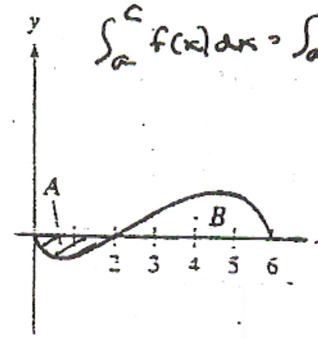
$2\int_2^5 f(x)\,dx+\int_2^5 3\,dx = 2\int_2^5 f(x)\,dx+9=17$ so $\int_2^5 f(x)\,dx=\boxed{4}$

3. Region A has an area of 1.5, and $\int_0^6 f(x)\,dx=3.5$. Find:

(a) $\int_2^6 f(x)\,dx = \int_0^6 f(x)\,dx-\int_0^2 f(x)\,dx$

$= 3.5-(-1.5)=\boxed{5}$

(b) $\int_0^6 |f(x)|\,dx = 1.5+5=\boxed{6.5}$

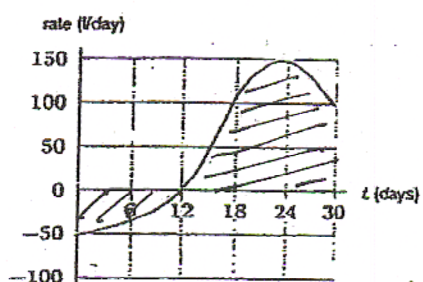


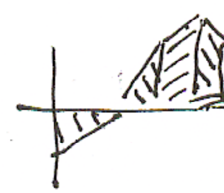
$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx+\int_b^c f(x)\,dx$

$\int_0^2 f(x)\,dx=-1.$

$\int_0^2 |f(x)|\,dx=1.$

4. The graph on the right shows the rate of change of the quantity of water in a water tower, in liters per day, during the month of April. If the tower has 12,000 liters of water in it on April 1, estimate the quantity of water in the tower on April 30.





$\approx 12{,}000-\frac{1}{2}(12)(50)+\frac{1}{2}(6)(100)$
$+\frac{1}{2}(6)(100+150)+\frac{1}{2}(6)(100+150) = \boxed{13{,}500 \text{ liters}}$

Answers will vary

Calc 5. A cup of coffee at 90° C is put into a 20° C room when $t=0$. The coffee's temperature is changing at a rate of $r(t)=-7e^{-0.3t}$ °C per minute, with t in minutes. Estimate the coffee's temperature when $t=10$.

Temp. $= 90-\int_0^{10} 7e^{-0.3t}\,dt = \boxed{67.828°C}$

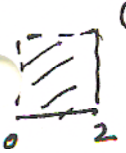
Calc 6. Water is pumped out of a holding tank at a rate of $5-5e^{-0.12t}$ liters/minute, where t is in minutes since the pump is started. If the holding tank contains 1000 liters of water when the pump is started, how much water does it hold one hour later?

Amt. of water $=1000-\int_0^{60}(5-5e^{-0.12t})\,dt = \boxed{741.636 \text{ liters}}$

7. Given the values of the derivative $f'(x)$ in the table and that $f(0)=100$, estimate $f(x)$ for $x=2, 4, 6$. Use a right Riemann sum.

$\int_0^2 f'(x)\,dx = f(2)-f(0)$

(2,18)



x	0	2	4	6
$f'(x)$	10	18	23	25

$\int_0^2 f'(x)\,dx$

$f(2)\approx 100+2(18)=\boxed{136}$

$f(4)\approx 136+2(23)=\boxed{182}$

$f(6)\approx 182+2(25)=\boxed{232}$

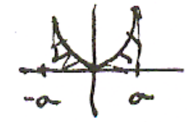
TURN->>>

$\int_0 f(x+2)\,dx =$

8. Consider the function f that is continuous on the interval $[-5,5]$ and for which $\int_0^5 f(x)\,dx = 4.$ Evaluate:

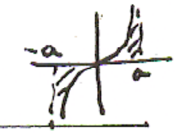
(a) $\int_0^5 (f(x)+2)\,dx = 4 + (2)(5) = \boxed{14}$

(c) $\int_{-5}^5 f(x)\,dx$ (f is even) $= \boxed{8}$

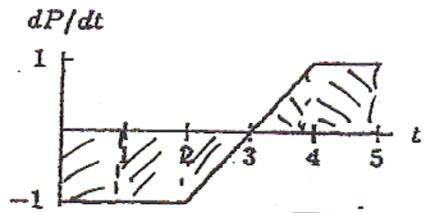


(b) $\int_{-2}^3 f(x+2)\,dx = \boxed{4}$

(d) $\int_{-5}^5 f(x)\,dx$ (f is odd) $= \boxed{0}$



9. Use the figure on the right and the fact that $P(0)=2$ to find values of P when $t = 1, 2, 3, 4,$ and 5.



$P(1) = 2 - (1)(1) = \boxed{1}$

$P(2) = 1 - (1)(1) = \boxed{0}$

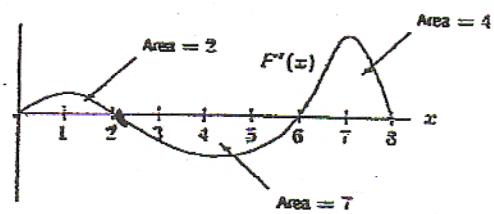
$P(3) = 0 - \frac{1}{2}(1)(1) = \boxed{-\frac{1}{2}}$

$P(4) = -\frac{1}{2} + \frac{1}{2}(1)(1) = \boxed{0}$

$P(5) = 0 + (1)(1) = \boxed{1}$

10. Use the figure on the right and the fact that $F(2)=3$ to sketch the graph of $F(x)$. Label the values of at least four points.

x	F(x)
0	1
6	-4
8	0
2	3

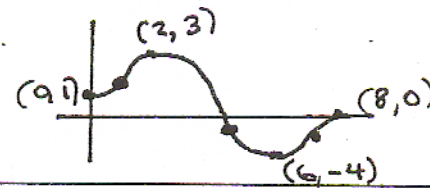


$F(0) = 3 - 2 = \boxed{1}$

$F(6) = 3 + {}^-7 = \boxed{-4}$

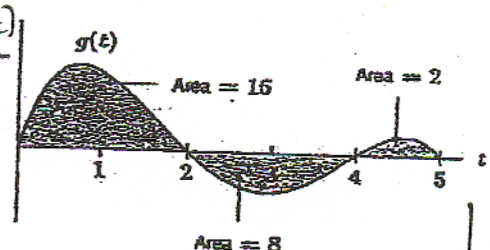
$F(8) = -4 + 4 = \boxed{0}$ and $(4\frac{1}{2}, ?)$

F is CU on $(0,1)$ & CD on $(1, 4\frac{1}{2})$ & $(7, 8)$.



11. Using the figure on the right, sketch a graph of an antiderivatives $G(t)$ of $g(t)$ satisfying $G(0)=5$. Label each critical point of $G(t)$ with its coordinates.

t	G(t)
0	5
2	21
4	13
5	15

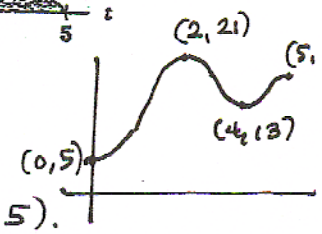


$G(2) = 5 + 16 = \boxed{21}$

$G(4) = 21 + {}^-8 = \boxed{13}$

$G(5) = 13 + 2 = \boxed{15}$

G is CU on $(0, \frac{2}{3})$ and $(3, 4\frac{1}{2})$ and CD on $(\frac{2}{3}, 3)$ and $(4\frac{1}{2}, 5)$.



Calc

12. Find the value of $F(1)$, where $F'(x) = e^{-x^2}$ and $F(0)=2$.

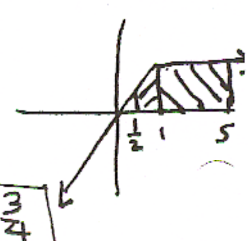
$F(1) = 2 + \int_0^1 e^{-x^2}\,dx = \boxed{2.747}$

$\int e^u\,du =$ We have ~~not~~ had this!

13. Given $f(x) = \begin{cases} 2x, & x \le 1 \\ 2, & x > 1 \end{cases}$. Evaluate: $\int_{1/2}^5 f(x)\,dx. = \int_{\frac{1}{2}}^{1} 2x\,dx + \int_1^5 2\,dx$

$= [x^2]_{\frac{1}{2}}^{1} + [2x]_1^5$

$= (1 - \frac{1}{4}) + (10 - 2) = \boxed{8\frac{3}{4}}$



14. A bowl of soup is placed on the kitchen counter to cool. The temperature of the soup is given in the table below.

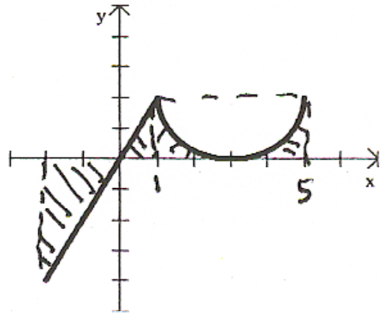
Time t (minutes)	0	5	8	12
Temperature $T(x)$ (°F)	105	99	97	93

(a) Find $\int_0^{12} T'(x)\,dx$. $= T(12) - T(0) = 93 - 105 = \boxed{-12° \text{ F}}$

(b) Find the average rate of change of $T(x)$ over the time interval $t = 5$ to $t = 8$ minutes.

$$\frac{T(8)-T(5)}{8-5} = \frac{97-99}{3} = \boxed{-\tfrac{2}{3}\ °\text{F/min.}}$$

15. The graph of f' which consists of a line segment and a semicircle, is shown on the right. Given that $f(1) = 4$, find:



(a) $f(-2) = 4 - \left(\tfrac{1}{2}(1)(2) - \tfrac{1}{2}(4)(2)\right)$
$= 4 - (1 - 4) = 7$

(b) $f(5) = 4 + ((4)(2) - 2\pi)$
$= \boxed{12 - 2\pi}$

Rectangle − Semicircle

16. (Multiple Choice) If f and g are continuous functions such that $g'(x) = f(x)$ for all x, then $\int_2^3 f(x)\,dx =$ $\int_2^3 g'(x)\,dx = g(3) - g(2)$

(A) $g'(2) - g'(3)$ (B) $g'(3) - g'(2)$ (C) $g(3) - g(2)$
(D) $f(3) - f(2)$ (E) $f'(3) - f'(2)$

17. (Multiple Choice) If the function $f(x)$ is defined by $f(x) = \sqrt{x^3 + 2}$ and g is an antiderivatives of f such that $g(3) = 5$, then $g(1) =$

$\int_1^3 g'(x)\,dx = g(3) - g(1)$

Calc

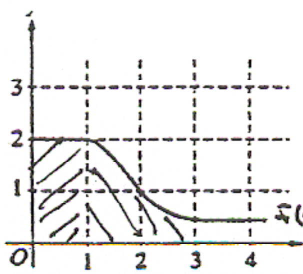
(A) -3.268 (B) -1.585 (C) 1.732 (D) 6.585 (E) 11.585

$g(1) = 5 - \int_1^3 \sqrt{x^3 + 2}\,dx = \boxed{-1.585}$

18. (Multiple Choice) The graph of f is shown in the figure at right.

If $\int_1^3 f(x)\,dx = 2.3$ and $F'(x) = f(x)$, then $F(3) - F(0) =$

(A) 0.3 (B) 1.3 (C) 3.3 (D) 4.3 (E) 5.3



$\int_1^3 F'(x)\,dx = F(3) - F(1) = 2.3$

$\int_0^1 F'(x)\,dx = 2$

$\int_0^3 F'(x)\,dx = 2 + 2.3 = \boxed{4.3 = F(3) - F(0)}$