CALCULUS
WORKSHEET 2 ON FUNCTIONS DEFINED BY INTEGRALS

Work the following on notebook paper.

1. Find the equation of the tangent line to the curve $y = F(x)$ where $F(x) = \int_1^x \sqrt[3]{t^2+7}\,dt$ at the point on the curve where $x = 1$.

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2. Suppose that $5x^3 + 40 = \int_c^x f(t)\,dt$.

(a) What is $f(x)$?

(b) Find the value of c.

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3. If $F(x) = \int_{-4}^x (t-1)^2(t+3)\,dt$, for what values of x is F decreasing? Justify your answer.

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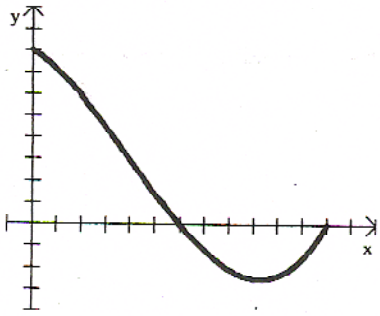
4. Let $H(x) = \int_0^x f(t)\,dt$ where f is the continuous function with domain [0, 12] shown on the right.



Graph of f

(a) Find $H(0)$.

(b) On what interval(s) of x is H increasing? Justify your answer.

(c) On what interval(s) of x is H concave up? Justify your answer.

(d) Is $H(12)$ positive or negative? Explain.

(e) For what value of x does H achieve its maximum value? Explain.

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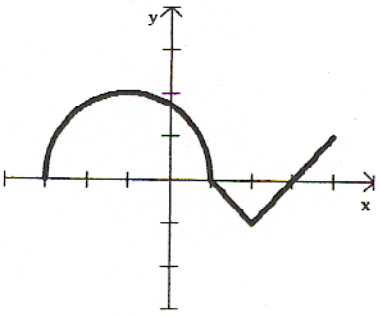
5. The graph of a function f consists of a semicircle and two line segments as shown on the right.



Graph of f

Let $g(x) = \int_1^x f(t)\,dt$.

(a) Find $g(1)$, $g(3)$, $g(-1)$.

(b) On what interval(s) of x is g decreasing? Justify your answer.

(c) Find all values of x on the open interval $(-3, 4)$ at which g has a relative minimum. Justify your answer.

(d) Find the absolute maximum value of g on the interval $[-3, 4]$ and the value of x at which it occurs. Justify your answer.

(e) On what interval(s) of x is g concave up? Justify your answer.

(f) For what value(s) of x does the graph of g have an inflection point? Justify your answer.

(g) Write an equation for the line tangent to the graph of g at $x = -1$.

TURN->>>

6. The graph of the function f, consisting of three line segments, is shown on the right.

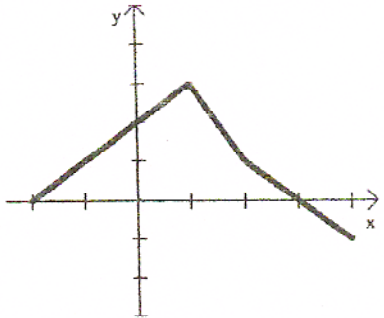
Let $g(x) = \int_1^x f(t)\,dt$.

(a) Find $g(2)$, $g(4)$, $g(-2)$.

(b) Find $g'(0)$ and $g'(3)$.

(c) Find the instantaneous rate of change of g with respect to x at $x = 2$.

(d) Find the absolute maximum value of g on the interval $[-2, 4]$. Justify your answer.



Graph of f

(e) The second derivative of g is not defined at $x = 1$ and at $x = 2$. Which of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.