Worksheet 2

① $F(x) = \int_1^x \sqrt[3]{t^2+7}\,dt$

$F(1) = \int_1^1 \sqrt[3]{t^2+7}\,dt = 0$ so $\underline{(1,0) = \text{point}}$

$F'(x) = \sqrt[3]{x^2+7}$ so $F'(1) = \sqrt[3]{8} = \underline{2}$

Tangent line : $\boxed{y - 0 = 2(x-1)}$

---

② $5x^3 + 40 = \int_c^x f(t)\,dt$

(a) $\frac{d}{dx}\left[5x^3 + 40\right] = \frac{d}{dx}\int_c^x f(t)\,dt$

$$\boxed{15x^2 = f(x)}$$

(b) $5x^3 + 40 = \int_c^x 15t^2\,dt = \left[5t^3\right]_c^x = 5x^3 - 5c^3$

$$40 = -5c^3$$

$$-8 = c^3$$

$$\boxed{-2 = c}$$

---

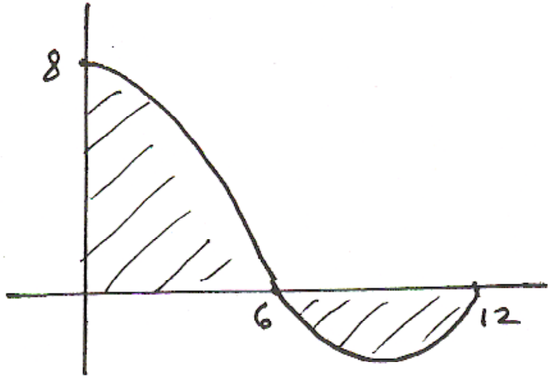
③ $F(x) = \int_{-4}^x (t-1)^2(t+3)\,dt$

$F'(x) = (x-1)^2(x+3)$

$F'(x)$	$-$		$+$		$+$
$F(x)$	$\searrow$	-3	$\nearrow$	1	$\nearrow$

$\boxed{\text{F is decreasing on } (-\infty,-3) \text{ because } F'(x) < 0 \text{ there.}}$

(4)



Graph of f

$$H(x) = \int_0^x f(t)\,dt$$

(a) $H(0) = \int_0^0 f(t)\,dt = \boxed{0}$

(b) $H'(x) = f(x)$

H is increasing on $(0, 6)$ because $\underline{H'(x) = f(x)}$ is positive.

(c) H is concave up on $(9\frac{1}{2}, 12)$ because $\underline{H'(x) = f(x)}$ is increasing there.

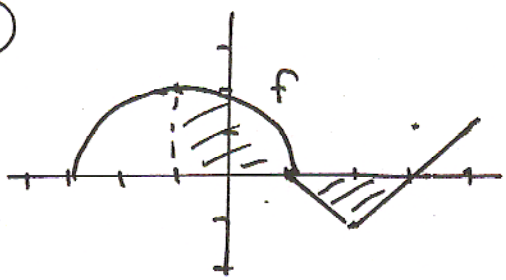
(d) $H(12)$ is positive because there is a greater area above the x-axis than below.

(e)

$H'(x)$		+		−	
$H(x)$	0	↗	6	↘	12

$H'(x) > 0$ on $(0, 6)$ and $H'(x) < 0$ on $(6, 12)$ so H has its maximum value at $x = 6$.

⑤



$g(x) = \int_1^x f(t)\,dt$

(a) $g(1) = \int_1^1 f(t)\,dt = \boxed{0}$

$g(3) = \int_1^3 f(t)\,dt = -\frac{1}{2}(2)(1) = \boxed{-1}$

$g(-1) = \int_1^{-1} f(t)\,dt$

$= -\int_{-1}^{1} f(t)\,dt$

$= -\frac{1}{4}\cdot 4\pi = \boxed{-\pi}$

(b) g is decreasing on (1, 3) because $g'(x) = f(x)$ is negative there.

(c) g has a rel. min. at $x = 3$ because $g'(x) = f(x)$ changes from negative to positive there.

(d)

x	g(x)
-3	-2π
1	0
3	-1
4	$-\frac{1}{2}$

g has its abs. max. value of 0 at $x = 1$

Candidates Test

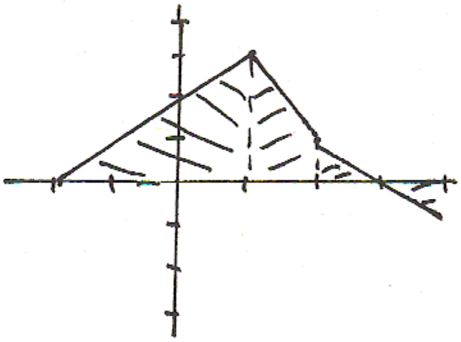
(e) g is concave up on (-3, -1) and (2, 4) because $g'(x) = f(x)$ is increasing there.

(f) g has an inflection point at $x = -1$ because $g'(x) = f(x)$ changes from increasing to decreasing there. g has an inflection point at $x = 2$ because $g'(x) = f(x)$ changes from decreasing to increasing there.

(g) $g(-1) = -\pi$ and $g'(-1) = f(-1) = 2$

Equation: $\boxed{y + \pi = 2(x+1)}$

⑥



Graph of f

$g(x) = \int_1^x f(t)\,dt$

(a) $g(2) = \int_1^2 f(t)\,dt = \frac{1}{2}(1)(3+1) = \boxed{2}$

$g(4) = \int_1^4 f(t)\,dt = 2 + \frac{1}{2} - \frac{1}{2} = \boxed{2}$

$g(-2) = \int_1^{-2} f(t)\,dt$

$= -\int_{-2}^{1} f(t)\,dt$

$= -\frac{1}{2}(3)(3) = \boxed{-\frac{9}{2}}$

(b) $g'(x) = f(x)$

$g'(0) = f(0) = \boxed{2}$ $\qquad$ $g'(3) = f(3) = \boxed{0}$

(c) $g'(2) = f(2) = \boxed{1}$

(d)

x	$g(x)$
-2	$-\frac{9}{2}$
3	$\frac{5}{2}$
4	2

g has its abs. maximum value of $\frac{5}{2}$.

(e) g has an inflection point at $x = 1$ because $g'(x) = f(x)$ changes from increasing to decreasing there. g does not have an inflection point at $x = 2$ because $g'(x) = f(x)$ is decreasing on $(1, 2)$ and continues to decrease on $(3, 4)$.