CALCULUS BC

WORKSHEET ON APPLICATIONS OF DIFFERENTIAL EQUATIONS

Work the examples on **notebook paper**.

Ex. The rate of change of the number of bears $N(t)$ is directly proportional to $1200 - N(t)$, where t is the time in years. When $t = 0$, the population is 300. If $N(4) = 600$, find $N(8)$ and find $\lim_{t \to \infty} N(t)$.

Newton's Law of Cooling: The rate of change in the temperature of an object is proportional to the difference between the temperature of the object and the temperature of the surrounding medium.

Ex. Suppose a room is kept at a constant temperature of 60° and an object cooled from 100° to 90° in ten minutes. How much longer will it take for its temperature to decrease to 80°?

Work the following on **notebook paper**. Use your calculator and give decimal answers correct to three decimal places.

1. A pie is removed from an oven at 450° and left to cool at a room temperature of 70°. After 30 minutes, the pie's temperature is 200°. How many minutes after being removed from the oven will the temperature of the pie be 100°?

2. A certain population increases at a rate proportional to the square root of the population. If the population goes from 2500 to 3600 in five years, what is the population at the end of t years?

3. Water leaks out of a barrel at a rate proportional to the square root of the depth of the water at that time. If the water level starts at 36 in. and drops to 35 in. in one hour, how long will it take for all of the water to leak out of the barrel?

4. A student studying a foreign language has 50 verbs to memorize. The rate at which the student can memorize these verbs is proportional to the number of verbs remaining to be memorized, $50 - y$, where the student has memorized y verbs. Assume that initially no verbs have been memorized and suppose that 20 verbs are memorized in the first 30 minutes.
   (a) How many verbs will the student memorize in two hours?
   (b) After how many hours will the student have only one verb left to memorize?

5. At time t, $t \geq 0$, the volume of a sphere is increasing at a rate proportional to the reciprocal of the radius. At $t = 0$, the radius of the sphere is 1, and at $t = 15$, the radius is 2.
   (a) Find the radius of the sphere as a function of t.
   (b) At what time t will the volume of the sphere be 27 times the volume at $t = 0$?

6. Let $P(t)$ represent the number of wolves in a population at time t years, where $t \geq 0$. The population $P(t)$ is increasing at a rate directly proportional to $800 - P(t)$, where the constant of proportionality is k.
   (a) If $P(0) = 500$, find $P(t)$ in terms of t and k.
   (b) If $P(2) = 700$, find k.
   (c) Find $\lim_{t \to \infty} P(t)$.