

$$(1) \frac{dy}{dt} = k(y-70)$$

$$(0, 450)$$

$$450 - 70 = C$$

$$380 = C$$

$$y - 70 = 380 e^{kt}$$

$$(30, 200)$$

$$200 - 70 = 380 e^{30k}$$

$$130 = 380 e^{30k}$$

$$\frac{\ln\left(\frac{13}{38}\right)}{30} = k$$

$$100 - 70 = 380 e^{kt}$$

$$30 = 380 e^{kt}$$

$$\frac{\ln\left(\frac{3}{38}\right)}{k} = t$$

$$t \approx \cancel{72.057} \boxed{71.011} \text{ minutes}$$

$$\boxed{\cancel{41.011} \text{ min}} \quad 41.011$$

$$(2) \frac{dP}{dt} = k\sqrt{P}$$

$$(0, 2500)$$

$$2\sqrt{2500} = C$$

$$100 = C$$

$$2\sqrt{P} = kt + 100$$

$$2\sqrt{P} = kt + C$$

$$(5, 3600)$$

$$2(60) = 5k + 100$$

$$120$$

$$-100$$

$$20 = 5k$$

$$4 = k$$

$$2\sqrt{P} = 4t + 100$$

$$\sqrt{P} = 2t + 50$$

$$P = (2t + 50)^2$$

③ $\frac{dh}{dt} = k\sqrt{h}$

$(0, 36)$

$12 = C$

$\frac{dh}{\sqrt{h}} = k dt$

$2\sqrt{h} = kt + 12$

$2\sqrt{35} = k + 12$

$2\sqrt{h} = kt + C$

$2\sqrt{35} - 12 = k$

$0 = kt + 12$

$\frac{-12}{k} = t$

$t = 71.496 \text{ hours}$

④ $\frac{dy}{dt} = k(50 - y)$

$(0, 0)$

$50 = C$

$\frac{dy}{50 - y} = k dt$

$50 - y = 50e^{-kt}$

$-\ln|50 - y| = kt + C$

$(\frac{1}{2}, 20)$

$30 = 50e^{-\frac{1}{2}k}$

$\ln|50 - y| = -kt + C$

$50 - y = Ce^{-kt}$

$\frac{\ln(\frac{3}{5})}{-\frac{1}{2}} = k$

$50 - y = 50e^{-kt}$

a) $50 - y = 50e^{-2k}$

b) $1 = 50e^{-k}$

$y = 50 - 50e^{-2k}$

$\ln(\frac{1}{50}) = -kt$

$y = 43.52 \text{ words}$

$\frac{\ln(\frac{1}{50})}{-k} = t$

So student has memorized 43 words.

$t = 3.829 \text{ hours}$

$$\textcircled{5} \quad \frac{dV}{dt} = k \cdot \frac{1}{r}$$

$$V = \frac{4}{3}\pi r^3$$
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$4\pi r^2 \frac{dr}{dt} = k \cdot \frac{1}{r}$$

$$r^3 \frac{dr}{dt} = \frac{1}{4\pi} \cdot k \, dt$$

$$\frac{1}{4} r^4 = \frac{k}{4\pi} t + C \quad \rightarrow \quad \frac{1}{4} = C$$

$$a) \quad r = 4 \sqrt[4]{\frac{k}{\pi} t + C}$$

$$r = 4 \sqrt[4]{\frac{k}{\pi} t + \frac{1}{4}}$$

$$r = 4 \sqrt[4]{\frac{21}{20} t + \frac{1}{4}}$$

$$2 = 4 \sqrt[4]{\frac{15k}{\pi} + \frac{1}{4}}$$

$$16 - \frac{1}{4} = \frac{15k}{\pi}$$

$$\frac{21\pi}{20} = k$$

$$b) \quad V = 36\pi = \frac{4}{3}\pi r^3$$

$$27 = r^3$$

$$3 = r$$

$$3 = 4 \sqrt[4]{\frac{21}{20} t + \frac{1}{4}}$$

$$\frac{81 - \frac{1}{4}}{\frac{21}{20}} = t$$

$$t = 76.9$$

$$6. \frac{dP}{dt} = k(800 - P)$$

$$\frac{dP}{800 - P} = k dt$$

$$-\ln |800 - P| = kt + C$$

$$\ln |800 - P| = -kt + C$$

$$800 - P = C e^{-kt}$$

$$300 = C$$

$$800 - P = 300 e^{-kt} \quad (a)$$

$$100 = 300 e^{-2k}$$

$$\frac{\ln \left| \frac{1}{3} \right|}{-2} = k \quad (b)$$

$$\lim_{t \rightarrow \infty} P(t) = 800$$