CALCULUS
WORKSHEET ON AVERAGE VALUE

Exercises

On problems 1 – 4,

(a) Find the average value of f on the given interval.

(b) Find c such that $f_{ave} = f(c)$.

(c) Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.

1. $f(x) = (x-3)^2$, $[2, 5]$

2. $f(x) = \sqrt{x}$, $[0, 4]$

3. $f(x) = 2\sin x - \sin 2x$, $[0, \pi]$

4. $f(x) = \dfrac{2x}{(1+x^2)^2}$, $[0, 2]$

5. If f is continuous and $\int_1^3 f(x)\,dx = 8$, show that f takes on the value 4 at least once on the interval $[1, 3]$.

6. Find the numbers b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.

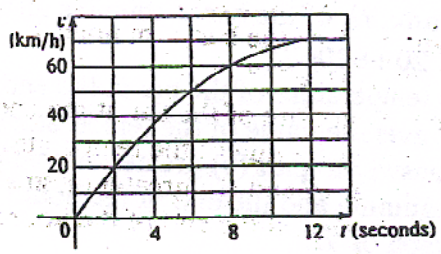
7. The table below gives values of a continuous function. Use the Midpoint Rule to estimate the average value of f on $[20, 50]$.

x	20	25	30	35	40	45	50
$f(x)$	42	38	31	29	35	48	60

8. The velocity graph of an accelerating car is shown on the right.

(a) Estimate the average velocity of the car during the first 12 seconds.

(b) At what time was the instantaneous velocity equal to the average velocity?



9. In a certain city the temperature (in °F) t hours after 9 AM was modeled by the function

$$T(t) = 50 + 14\sin\left(\frac{\pi t}{12}\right).$$

Find the average temperature during the period from 9 AM to 9 PM.

10. If a cup of coffee has temperature 95° C in a room where the temperature is 20° C, then, according to Newton's Law of Cooling, the temperature of the coffee after t minutes is

$$T(t)=20+75e^{-t/50}.$$

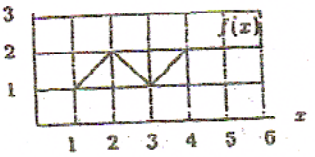
What is the average temperature of the coffee during the first half hour?

11. Suppose that $C(t)$ represents the daily cost of heating your house, measured in dollars per day, where t is time measured in days and $t=0$ corresponds to January 1, 2001. Interpret $\int_0^{90} C(t)\,dt$ and $\frac{1}{90-0}\int_0^{90} C(t)\,dt$.

12. Using the figure on the right,

(a) Find $\int_1^6 f(x)\,dx$.

(b) What is the average value of f on $[1, 6]$?



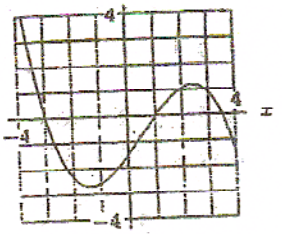
13. Using the figure on the right,

(a) Estimate $\int_{-3}^{3} f(x)\,dx$.

(b) Which of the following average values of $f(x)$ is larger?

(i) Between $x=-3$ and $x=3$

(ii) Between $x=0$ and $x=3$



14. A bar of metal is cooling from 1000° C to room temperature, 20° C. The temperature, H, of the bar t minutes after it starts cooling is given, in ° C, by

$$H=20+980e^{-0.1t}.$$

(a) Find the temperature of the bar at the end of one hour.

(b) Find the average value of the temperature over the first hour.

(c) Is your answer to part (b) greater or smaller than the average of the temperatures at the beginning and the end of the hour? Explain this in terms of the concavity of the graph of H.

15. The average value of $y=v(x)$ equals 4 for $1\le x\le 6$ and equals 5 for $6\le x\le 8$. What is the average value of $v(x)$ for $1\le x\le 8$?

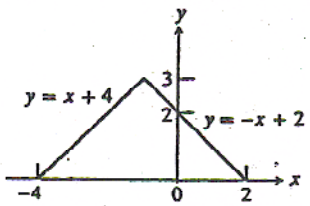
16. (a) Let $\int_0^3 f(x)dx = 6$. What is the average value of $f(x)$ on the interval $x = 0$ to $x = 3$?

(b) If $f(x)$ is even, what is $\int_{-3}^3 f(x)dx$? What is the average value of $f(x)$ on the interval $x = -3$ to $x = 3$?

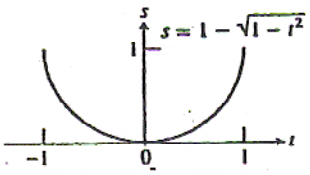
(c) If $f(x)$ is odd, what is $\int_{-3}^3 f(x)dx$? What is the average value of $f(x)$ on the interval $x = -3$ to $x = 3$?

In problems 27 – 30, find the average value of the function on the interval without integrating, by appealing to the geometry of the region between the graph and the x-axis.

17. $f(x) = \begin{cases} x+4, & -4 \le x \le -1 \\ -x+2, & -1 \le x \le 2 \end{cases}$ on $[-4, 2]$



18. $f(t) = 1 - \sqrt{1-t^2}$, $[-1, 1]$



19. $f(t) = \sin t$, $[0, 2\pi]$

20. $f(\theta) = \tan\theta$, $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$