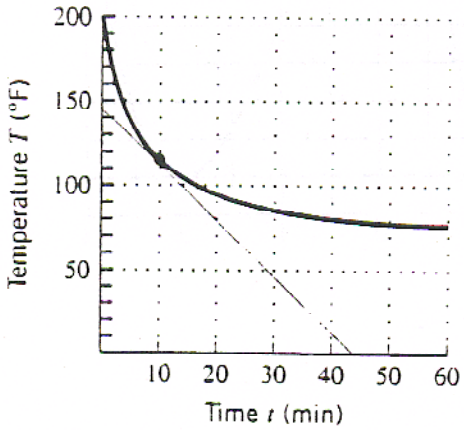
CALCULUS
WORKSHEET ON DERIVATIVES (from Graphs)

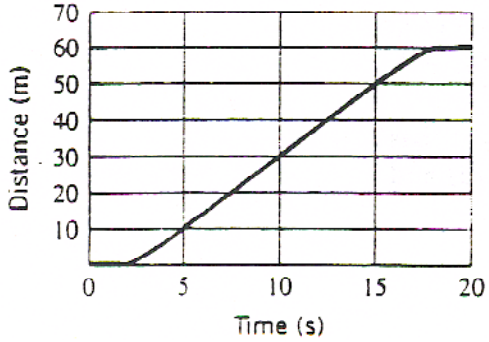
1. According to *Newton's Law of Cooling*, the rate of change of an object's temperature is proportional to the difference between the temperature of the object and that of the surrounding medium. The accompanying figure shows the graph of the temperature T (in degrees Fahrenheit) versus time t (in minutes) for a cup of coffee, initially with a temperature of 200° F, that is allowed to cool in a room with a constant temperature of 75° F.
   (a) Estimate T and dT/dt when $t = 10$ min.
   (b) Newton's Law of Cooling can be expressed as

$$\frac{dT}{dt} = k(T - T_0)$$

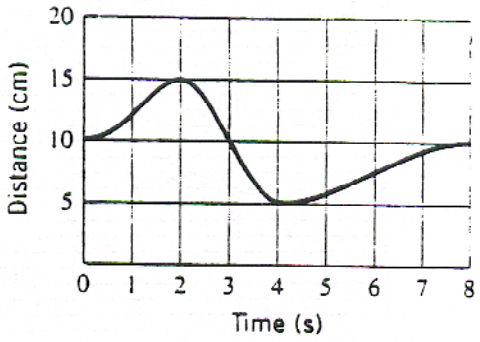
   where k is the constant of proportionality and T_0 is the temperature (assumed constant) of the surrounding medium. Use the results in part (a) to estimate the value of k.



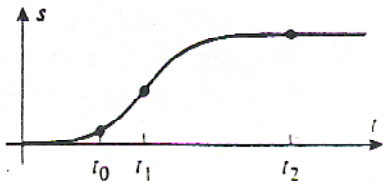
2. Figure 3.1.10 shows the position versus time curve for an elevator that moves upward a distance of 60 meters and then discharges its passengers.
   (a) Estimate the instantaneous velocity of the elevator at $t = 10$ seconds.
   (b) Sketch a velocity versus time curve for the motion of the elevator for $0 \le t \le 20$.



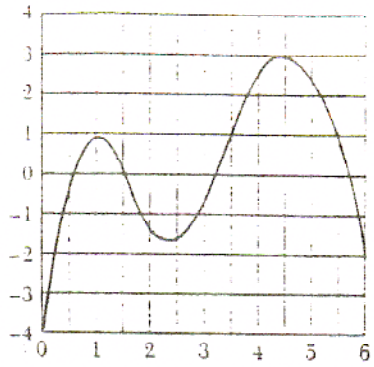
3. The accompanying figure shows the position versus time curve for a certain particle moving along a straight line. Estimate each of the following from the graph:
   (a) the average velocity over the interval $0 \le t \le 3$
   (b) the values of t at which the instantaneous velocity is zero
   (c) the values of t at which the instantaneous velocity is either a maximum or a minimum
   (d) the instantaneous velocity when $t = 3$ s.



4. The accompanying figure shows the position versus time curve for a certain particle moving on a straight line.
   (a) Is the particle moving faster at time t_0 or time t_2? Explain.
   (b) At the origin, the tangent is horizontal. What does this tell us about the initial velocity of the particle?
   (c) Is the particle speeding up or slowing down in the interval $[t_0, t_1]$? Explain.
   (d) Is the particle speeding up or slowing down in the interval $[t_1, t_2]$? Explain.



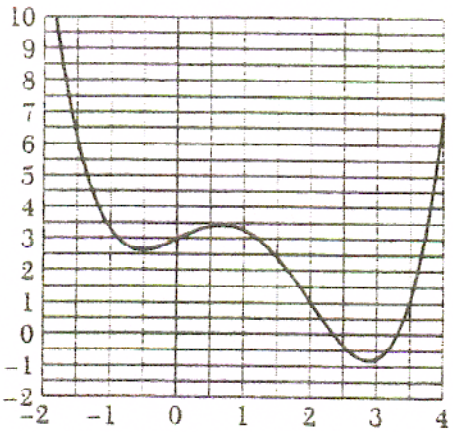
5. **Graph of f**



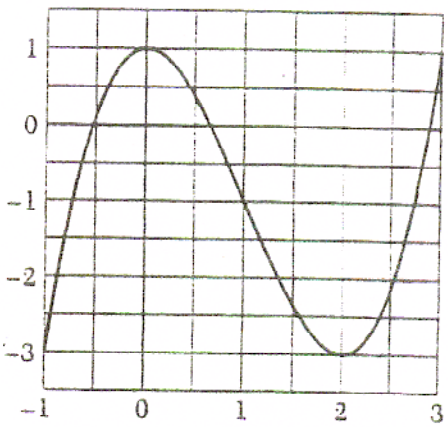
Check the entries in the table below and replace any that are wrong with reasonable estimates.

x	1	2	3	4	5	6
$f'(x)$	0	2	-6	3	−2	42

6. **Graph of f**



Graph of f'



Find equations for the lines tangent to f at $x = -1$, $x = 0$, $x = 1$, $x = 2$, and $x = 3$.

7. (a) Suppose that the line tangent to the curve $y = f(x)$ at $x = -2$ is described by the equation $y = 4x + 3$. Find $f(-2)$ and $f'(-2)$.

(b) The line tangent to f at $(5, 2)$ passes through the point $(0, 1)$. Find $f(5)$ and $f'(5)$.

(c) Suppose that the line tangent to the curve $y = f(x)$ at $x = 3$ passes through the points $(-2, 3)$ and $(4, -1)$. Find $f(3)$ and $f'(3)$.

(d) The line tangent to the curve $y = f(x)$ at $x = 2$ has slope -1 and crosses the x axis at $x = 5$. Find $f(2)$ and $f'(2)$.