# Worksheet

① $\frac{dy}{dx} = \frac{x^2}{y}$, $y = -5$ when $x = 3$

$$y\,dy = x^2\,dx$$
$$\frac{y^2}{2} = \frac{x^3}{3} + C$$
$$\frac{25}{2} = 9 + C$$
$$\frac{7}{2} = C$$
$$\frac{y^2}{2} = \frac{x^3}{3} + \frac{7}{2}$$
$$y^2 = \frac{2x^3}{3} + 7$$
$$y = \pm\sqrt{\frac{2x^3}{3} + 7}$$

Since $y(3) = -5$,

$$\boxed{y = -\sqrt{\frac{2x^3}{3} + 7}}$$

② $\frac{dy}{dx} = 6x^2y$ and $y(0) = 4$

$$\frac{dy}{y} = 6x^2\,dx$$
$$\ln|y| = 2x^3 + C_1$$
$$|y| = e^{2x^3 + C_1} = e^{2x^3}\cdot e^{C_1}$$
$$|y| = Ce^{2x^3} \quad (C = e^{C_1})$$
$$4 = C$$
$$|y| = 4e^{2x^3} \qquad |y| = \pm y$$

Since $y(0) = 4$,

$$\boxed{y = 4e^{2x^3}}$$

③ $\frac{dy}{dx} = \frac{1}{y^2}$ and $y(0) = 4$

$$y^2\,dy = 1\,dx$$
$$\frac{y^3}{3} = x + C$$
$$\frac{64}{3} = C$$
$$\frac{y^3}{3} = x + \frac{64}{3}$$
$$y^3 = 3x + 64$$
$$\boxed{y = \sqrt[3]{3x + 64}}$$

④ $\frac{dy}{dx} = \frac{1+x}{\sqrt{y}}$ and $y(2) = 9$

$$\sqrt{y}\,dy = (1 + x)\,dx$$
$$\frac{2}{3}y^{\frac{3}{2}} = x + \frac{x^2}{2} + C$$
$$18 = 2 + 2 + C$$
$$14 = C$$
$$\frac{2}{3}y^{\frac{3}{2}} = x + \frac{x^2}{2} + 14$$
$$y^{\frac{3}{2}} = \frac{3}{2}x + \frac{3}{4}x^2 + 21$$
$$\boxed{y = \left(\frac{3}{2}x + \frac{3}{4}x^2 + 21\right)^{2/3}}$$

⑤ $\frac{dy}{dx} = -xy^2$ and $y(1) = -0.25$

$\int y^{-2}\,dy$ $\quad \frac{dy}{y^2} = -x\,dx$

$-\frac{1}{y}$ $\quad -\frac{1}{y} = -\frac{x^2}{2} + C$

$4 = -\frac{1}{2} + C$

$\frac{9}{2} = C$

$-\frac{1}{y} = -\frac{x^2}{2} + \frac{9}{2} = \frac{9-x^2}{2}$

$\frac{1}{y} = \frac{x^2-9}{2}$

$\boxed{y = \frac{2}{x^2-9}}$

⑥ $\frac{dy}{dx} = \frac{4\sqrt{y}\ln x}{x}$, $y(e) = $

$\int y^{-\frac{1}{2}}\,dy$ $\quad \frac{dy}{\sqrt{y}} = \frac{4\ln x}{x}\,dx$

$y^{-\frac{1}{2}}\,dy = \frac{4\ln x}{x}\,dx$

$u = \ln x$ $\quad 4\int u\,du = \frac{4u^2}{2} + C = 2u^2 + C$

$du = \frac{1}{x}\,dx$

$2y^{\frac{1}{2}} = 2(\ln x)^2 + C$

$6 = 2 + C$

$4 = C$

$2\sqrt{y} = 2(\ln x)^2 + 4$

$\sqrt{y} = (\ln x)^2 + 2$

$\boxed{y = ((\ln x)^2 + 2)^2}$

⑦ $\frac{dy}{dx} = 4x^3y \quad (0, 7)$

$\frac{dy}{y} = 4x^3\,dx$

$\ln|y| = x^4 + C_1$

$|y| = e^{x^4 + C_1} = e^{x^4} \cdot e^{C_1}$

$|y| = Ce^{x^4} \quad (C = e^{C_1})$

$7 = C$

$|y| = 7e^{x^4}$

Since $y(0) = 7$,

$\boxed{y = 7e^{x^4}}$

⑧ Slope is $\frac{y^2}{x^3}$ $\quad (1, 1)$

$\frac{dy}{dx} = \frac{y^2}{x^3}$

$\int y^{-2}$ $\quad \frac{dy}{y^2} = \frac{dx}{x^3}$ $\quad \int x^{-3}$

$-\frac{1}{y} = -\frac{1}{2x^2} + C$

$-1 = -\frac{1}{2} + C$

$-\frac{1}{2} = C$

$-\frac{1}{y} = -\frac{1}{2x^2} - \frac{1}{2}$

$\frac{1}{y} = \frac{1+x^2}{2x^2}$

$\boxed{y = \frac{2x^2}{1+x^2}}$

(9) $\frac{dy}{dt} = -3y$ and $y = 1$ when $t = 0$

$\frac{dy}{y} = -3\,dt$

$\ln|y| = -3t + C_1$

$|y| = e^{-3t+C_1} = e^{-3t} \cdot e^{C_1}$

$|y| = Ce^{-3t}$ $(C = e^{C_1})$

$1 = C$

$|y| = e^{-3t}$

Since $y(0) = 1$,

Particular solution $\boxed{y = e^{-3t}}$

When $y = \frac{1}{3}$, $\frac{1}{3} = e^{-3t}$

$\ln\frac{1}{3} = -3t$

$\boxed{t = -\frac{\ln\frac{1}{3}}{3} \text{ or } t = \frac{\ln 3}{3}}$

---

(10) $\frac{dy}{dx} = y\cos x$, $y = 3$ when $x = 0$

$\frac{dy}{y} = \cos x\,dx$

$\ln|y| = \sin x + C_1$

$|y| = e^{\sin x + C_1} = e^{\sin x} \cdot e^{C_1}$

$|y| = Ce^{\sin x}$ $(C = e^{C_1})$

$3 = C$

$|y| = 3e^{\sin x}$

Since $y(0) = 3$,

$\boxed{y = 3e^{\sin x}}$

On your TI-89, find deSolve under F3:

deSolve(y' = y·cos x, x, y)

should give $\boxed{y = Ce^{\sin x}}$.

deSolve(y' = y·cos x and y(0) = 3, x, y)

should give $\boxed{y = 3e^{\sin x}}$

(11) $f'(x) = 2f(x), \quad f(2) = 1$

$$\frac{dy}{dx} = 2y$$

$$\frac{dy}{y} = 2dx$$

$$\ln|y| = 2x + C_1$$

$$|y| = e^{2x+C_1} = e^{2x} \cdot e^{C_1}$$

$$|y| = Ce^{2x} \quad (C = e^{C_1})$$

$$1 = Ce^4$$

$$e^{-4} = C$$

$$|y| = e^{-4} \cdot e^{2x}$$

$$|y| = e^{2x-4}$$

Since $y(2) = 1$,

$$\boxed{y = e^{2x-4}} = \frac{e^{2x}}{e^4}$$

(12) $\frac{dy}{dx} = 2y^2$; $y = -1$ when $x = 1$

$$\frac{dy}{y^2} = 2dx \quad y^{-2}dy = 2$$

$$-\frac{1}{y} = 2x + C$$

$$\int y^{-2}dy = \frac{y^{-1}}{-1} = -\frac{1}{y}$$

$$1 = 2 + C$$

$$-1 = C$$

$$-\frac{1}{y} = 2x - 1$$

$$\int \frac{1}{y}dy = \ln|y|$$

$$\frac{1}{y} = 1 - 2x$$

$$\boxed{y = \frac{1}{1-2x}}$$

When $x = 2$, $\boxed{y = -\frac{1}{3}}$ $\boxed{B}$

(13) $\frac{dy}{dx} = x^2 y$

$$\frac{dy}{y} = x^2 dx$$

$$\ln|y| = \frac{x^3}{3} + C_1$$

$$|y| = e^{\frac{x^3}{3} + C_1} = e^{\frac{x^3}{3}} \cdot e^{C_1}$$

$$|y| = Ce^{\frac{x^3}{3}} \quad (C = e^{C_1})$$

so $y$ could be $\boxed{2e^{\frac{x^3}{3}}}$ $\boxed{C}$

(14) The rate of change of N with respect to s is proportional to 250 − N.

$$\frac{dN}{ds} = k(250 - N)$$

$$\frac{dN}{250 - N} = k\,ds$$

$$\boxed{\begin{aligned} u &= 250 - N \\ du &= -dN \end{aligned}}$$

$$-\ln|250 - N| = ks + C_1$$

$$\ln|250 - N| = -ks - C_1$$

$$\pm(250 - N) = e^{-ks - C_1} = e^{-ks} \cdot e^{-C_1}$$

$$250 - N = \pm Ce^{-ks} \qquad (C = e^{-C_1})$$

$$\boxed{250 \pm Ce^{-ks} = N} \qquad \boxed{N = 250 \pm Ce^{-ks}}$$

---

(15) The rate of change of R with respect to t is inversely proportional to the square root of R.

$$\frac{dR}{dt} = \frac{k}{\sqrt{R}}$$

$$\sqrt{R}\,dR = k\,dt$$

$$\frac{2}{3}R^{3/2} = kt + C_1$$

$$R^{3/2} = \frac{3}{2}kt + \underline{\frac{3}{2}C_1} \qquad \text{Let } \underline{C = \frac{3}{2}C_1}$$

$$R^{3/2} = \frac{3}{2}kt + C$$

$$\boxed{R = \left(\frac{3}{2}kt + C\right)^{2/3}}$$

OR $\boxed{R = \left(\frac{3}{2}kt + \frac{3}{2}C_1\right)^{2/3}}$

(16) The rate of change of $y$ with respect to $x$ varies jointly with $x$ and $L-y$.

$$\frac{dy}{dx} = kx(L-y)$$

$$\frac{dy}{L-y} = kx\,dx$$

$$u = L - y$$
$$du = -dy$$

$$-\ln|L-y| = \frac{kx^2}{2} + C_1$$

$$\ln|L-y| = -\frac{kx^2}{2} - C_1$$

$$\pm(L-y) = e^{-\frac{kx^2}{2} - C_1} = e^{-\frac{kx^2}{2}} \cdot e^{-C_1}$$

$$L - y = \pm Ce^{-\frac{kx^2}{2}} \quad (C = e^{-C_1})$$

$$\boxed{L \pm Ce^{-\frac{kx^2}{2}} = y}$$