CALCULUS BC
WORKSHEET ON EULER'S METHOD

Work the following on notebook paper, showing all steps.

1. (a) Given the differential equation $\frac{dy}{dx} = x+2$ and $y(0)=3$. Find an approximation for $y(1)$ by using Euler's method with two equal steps. Sketch your solution.

   (b) Solve the differential equation $\frac{dy}{dx} = x+2$ with the initial condition $y(0)=3$, and use your solution to find $y(1)$.

   (c) The error in using Euler's Method is the difference between the approximate value and the exact value. What was the error in your answer? How could you produce a smaller error using Euler's Method?

2. Suppose a continuous function f and its derivative f' have values that are given in the following table. Given that $f(2)=5$, use Euler's Method with two steps of size $\Delta x = 0.5$ to approximate the value of $f(3)$.

x	2.0	2.5	3.0
$f'(x)$	0.4	0.6	0.8
$f(x)$	5		

3. Given the differential equation $\frac{dy}{dx} = \frac{1}{x+2}$ and $y(0)=1$. Find an approximation of $y(1)$ using Euler's Method with two steps and step size $\Delta x = 0.5$.

4. Given the differential equation $\frac{dy}{dx} = x+y$ and $y(1)=3$. Find an approximation of $y(2)$ using Euler's Method with two equal steps.

5. The curve passing through $(2, 0)$ satisfies the differential equation $\frac{dy}{dx} = 4x+y$. Find an approximation to $y(3)$ using Euler's Method with two equal steps.

6. Assume that f and f' have the values given in the table. Use Euler's Method with two equal steps to approximate the value of $f(4.4)$.

x	4	4.2	4.4
$f'(x)$	-0.5	-0.3	-0.1
$f(x)$	2		

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7. The table gives selected values for the derivative of a function f on the interval $-2 \le x \le 2$. If $f(-2)=3$ and Euler's method with a step-size of 1.5 is used to approximate $f(1)$, what is the resulting approximation?

x	$f'(x)$
-2	-0.8
-1.5	-0.5
-1	-0.2
-0.5	0.4
0	0.9
0.5	1.6
1	2.2
1.5	3
2	3.7

8. Let $y=f(x)$ be the particular solution to the differential equation $\frac{dy}{dx}=x+2y$ with the initial condition $f(0)=1$. Use Euler's method, starting at $x=0$ with two steps of equal size, to approximate $f(-0.6)$.