CALCULUS BC
WORKSHEET ON IMPLICIT DIFFERENTIATION

Work these on notebook paper. Show all work, and circle your answers.

On problems 1 – 3, find $\frac{dy}{dx}$.

1. $x^3 + xy + y^3 = 1$ 2. $y - x\sin y = 3$ 3. $x + \tan(xy) = 0$

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4. If $y = xy + x^2 + 1$, find $\frac{dy}{dx}$ when $x = -1$.

5. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, given $y^2 + 2y = 2x + 1$.

6. If $x^3 + y^3 = 8$, show that the second derivative of y with respect to x is $-\frac{16x}{y^5}$.

7. Consider the curve defined by the equation $y + \cos y = x + 1$ for $0 \le y \le 2\pi$.
   (a) Find $\frac{dy}{dx}$ in terms of y.
   (b) Write an equation for each vertical tangent to the curve.
   (c) Find $\frac{d^2y}{dx^2}$ in terms of y.

8. Consider the curve $y^2 = 4 + x$ and the chord AB joining points $A(-4, 0)$ and $B(0, 2)$ on the curve. Find the x- and y-coordinates of the point on the curve where the tangent line is parallel to chord AB.

9. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

   Find $\frac{d^2y}{dx^2}$, and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

10. Consider the curve given by $xy^2 - x^3y = 6$.
    (a) Find $\frac{dy}{dx}$.
    (b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line to the curve at each of these points.
    (c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

TURN->>>

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Evaluate the following by recognizing that the given limit represents a derivative.

Ex. $\lim_{h\to0}\frac{\sin\left(\frac{\pi}{6}+h\right)-\frac{1}{2}}{h}=\lim_{h\to0}\frac{\sin\left(\frac{\pi}{6}+h\right)-\sin\frac{\pi}{6}}{h}=f'\left(\frac{\pi}{6}\right)$ where $f(x)=\sin x$.

Using our shortcuts, $f'(x)=\cos x$ so $f'\left(\frac{\pi}{6}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$.

Show the steps that lead to your answer, using the example above as a model.

11. $\lim_{h\to0}\frac{\tan\left(\frac{\pi}{4}+h\right)-1}{h}$

12. $\lim_{h\to0}\frac{5(x+h)^2-5x^2}{h}$

13. $\lim_{h\to0}\frac{(h+2)^6-64}{h}$

14. $\lim_{h\to0}\frac{\tan(3(x+h))-\tan(3x)}{h}$

15. $\lim_{x\to2}\frac{2x^3-16}{x-2}$

16. $\lim_{x\to\frac{\pi}{2}}\frac{\sin x-1}{x-\frac{\pi}{2}}$

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17. If f is a differentiable function, then $f'(a)$ is given by which of the following?

I. $\lim_{h\to0}\frac{f(a+h)-f(a)}{h}$ II. $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ III. $\lim_{x\to a}\frac{f(x+h)-f(x)}{h}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

Suppose f and g are differentiable functions with the values shown in the following table. For each of the following functions, find $h'(2)$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	5	5	e	$\sqrt{2}$
5	2	8	π	7

18. $h(x)=f(x)+g(x)$

19. $h(x)=f(x)g(x)$

20. $h(x)=\frac{f(x)}{g(x)}$

21. $h(x)=f(g(x))$

22. $h(x)=g(f(x))$

23. $h(x)=f(f(x))$

24. $h(x)=(f(x))^3$