CALCULUS BC

WORKSHEET ON DERIVATIVES (meaning)

Work the following on **notebook paper.** Use your graphing calculator to calculate derivatives. Give decimal answers correct to **three** decimal places.

1. The temperature T, in degrees Fahrenheit, of a frozen pizza placed in a hot oven is given by $T = f(x)$, where t is the time in minutes since the pizza was put into the oven.

(a) What is the sign of $f'(t)$? Why?

(b) Explain the meaning of the statement $f'(20) = 2$. (Be sure to include units)

2. Let $f(t)$ be the number of centimeters of rainfall that has fallen since midnight, where t is the time in hours. Interpret the following in practical terms, giving units.

(a) $f(10) = 3.1$

(b) $f^{-1}(10) = 16$

(c) $f'(8) = 0.4$

3. Let $g(v)$ be the fuel consumption, in miles per gallon, of a car going at v miles per hour.

(a) What are the units of $g'(55)$?

(b) What is the practical meaning of the statement $g'(55) = -0.54$?

4. Let W be the amount of water, in gallons, in a bathtub at time, t, in minutes.

(a) What are the meaning and units of $\frac{dW}{dt}$?

(b) Suppose the bathtub is full of water at time t_0, so that $W(t_0) > 0$. Subsequently, at time $t_p > t_0$, the plug is pulled. Is $\frac{dW}{dt}$ positive, negative, or zero:

(i) For $t_0 < t < t_p$?

(ii) After the plug is pulled?

(iii) When all the water has drained from the tub?

5. The number of bacteria after t hours in a controlled laboratory experiment is $n = f(t)$.

(a) What is the meaning of the derivative $f'(5)$? What are its units?

(b) Suppose there is an unlimited amount of space and nutrients for the bacteria. Which do you think is larger, $f'(5)$ or $f'(10)$? If the supply of nutrients is limited, would that affect your conclusion? Explain.

6. A small rock is tied to an inflated balloon, and then the rock and balloon are thrown into the air. While the rock and the balloon are moving, the height of the rock is given by $h(t) = -x^2 + 8x + 2$ where $h(x)$ is in feet above the ground at time x seconds after the rock is thrown.

(a) Find the value of $h(6)$, and explain its meaning.

(b) Find the value of $h'(6)$, and explain its meaning.

7. Lee Per attaches himself to a strong bungee cord and jumps off a bridge. At time $t - 3$ sec., the cord first becomes taunt. From that time on, Lee's distance, y, in feet from the river below the bridge is given by the equation $y = 90 - 80\sin(1.2(t-3))$.

(a) Find the value of $y(15)$, and explain its meaning.

(b) Find the value of $y'(15)$, and explain its meaning.

8. The depth of water in a tidal basin can be represented by the formula $h(t) = -\cos\left(\frac{\pi t}{12}\right) + 4$, where t is the time in hours starting at midnight and $h(t)$ is measured in feet.

(a) Find the value of $h(17)$, and explain its meaning.

(b) Find the value of $h'(17)$, and explain its meaning.

9. A pebble is stuck in the tread of a car tire. As the wheel turns, the distance, y, in inches, between the pebble and the road at various times, t, in seconds, is given by the table below.

t (sec)	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
y (in.)	0.63	0.54	0.45	0.34	0.22	0.00	0.22	0.34	0.45

(a) Estimate the value of $y(1.45)$, and explain its meaning. Show your computations.

(b) Estimate the value of $y'(1.45)$, and explain its meaning. Show your computations.

10. Let $y(t)$ be the temperature in degrees Fahrenheit in Houston t hours after midnight on June 1, 2005. The table shows values of this function recorded every two hours.

t (hrs)	0	2	4	6	8	10	12	14
y (°F)	74	75	72	76	81	88	90	92

(a) Estimate the value of $y(9)$, and explain its meaning. Show your computations.

(b) Estimate the value of $y'(9)$, and explain its meaning. Show your computations.

11. Life expectancy improved dramatically in the 20th century. The table below gives values of $E(t)$, the life expectancy at birth (in years) of a male born in the year t in the United States.

t	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000
$E(t)$	48.3	51.1	55.2	57.4	62.5	65.6	66.6	67.1	70.0	71.8	74.1

(a) Estimate the value of $E(1935)$, and explain its meaning. Show your computations.

(b) Estimate the value of $E'(1935)$, and explain its meaning. Show your computations.