CALCULUS

WORKSHEET ON NEWTON'S METHOD & TANGENT LINE APPROXIMATION

Use Newton's Method to estimate the zeros correct to three decimal places.

1. $f(x) = x^3 + x - 1$ $\quad x_1 = 0.5$

2. $f(x) = x^4 - 10x^2 - 11$ $\quad x_1 = 3.5$

3. $f(x) = x + \sin(x+1)$ $\quad x_1 = -0.5$

Use Newton's Method to estimate the points of intersection correct to three decimal places.

4. $f(x) = 2x + 1, \quad g(x) = \sqrt{x+4}$ $\quad x_1 = 0.6$

5. $f(x) = x^2, \quad g(x) = \cos x$ $\quad x_1 = \frac{\pi}{4}$

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6. (a) For $f(x) = 0.2x^4$, find an equation of the linear function that best fits f at $x = 3$.

(b) Use the tangent line equation you found in (a) to approximate $f(3.001)$.

(c) Find $f(3.001)$ by using the function $f(x)$. What is the error in your linear approximation?

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7. (a) For $g(x) = \sec x$, find an equation of the linear function that best fits g at $x = \frac{\pi}{3}$.

(b) Use the tangent line equation you found in (a) to approximate $g\left(\frac{\pi}{3} - .01\right)$.

(c) Find $g\left(\frac{\pi}{3} - .01\right)$ by using the function $g(x)$. What is the error in your linear approximation?

8. Let $f(x) = (1 + \tan x)^{1/2}$

(a) Write an equation for the line tangent to the graph of f at the point where $x = 0$.

(b) Use the equation found in part (a) to approximate $f(0.02)$.

(c) Find $f(0.02)$ by using the function $f(x)$. What is the error in your linear approximation? What does the error tell you about the sign of $f''(x)$?

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9. Consider the curve defined by $-8x^2 + 5xy + y^3 = -149$.

(a) Find $\frac{dy}{dx}$.

(b) Write an equation for the line tangent to the curve at point (4, -1).

(c) There is a number k so that the point (4.2, k) is on the curve. Using the tangent line found in part (b), approximate the value of k.

(d) Write an equation that can be solved to find the actual value of k so that the point (4.2, k) is on the curve, and then use it to solve for k.

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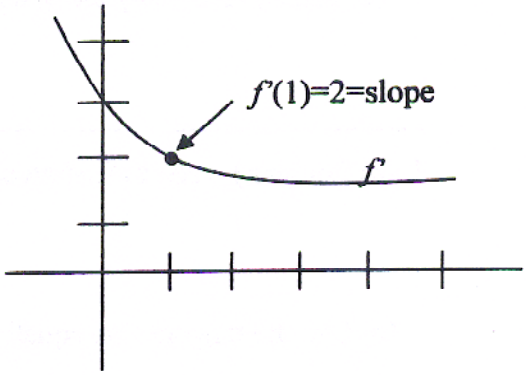
10. Suppose that the only information you have about a function f is that $f(1) = 5$ and the graph of f' is given on the right.

(a) Use a linear approximation to estimate $f(0.9)$ and $f(1.2)$



(b) Are your estimates in part (a) too large or too small? Explain.