CALCULUS
WORKSHEET ON RIEMANN SUMS AND TRAPEZOIDAL RULE

1. A table of values for $f(t)$ is given.

t	0	20	40	60	80	100	120
$f(t)$	1.2	2.8	4.0	4.7	5.1	5.2	4.8

(a) Estimate $\int_0^{120} f(t)$ by using a left Riemann sum with six subintervals.

(b) Estimate $\int_0^{120} f(t)$ by using a right Riemann sum with six subintervals.

(c) Estimate $\int_0^{120} f(t)$ by using a midpoint sum with three subintervals.

(d) Estimate $\int_0^{120} f(t)$ by using the trapezoidal rule with three subintervals.

2. A table of values for $g(t)$ is given.

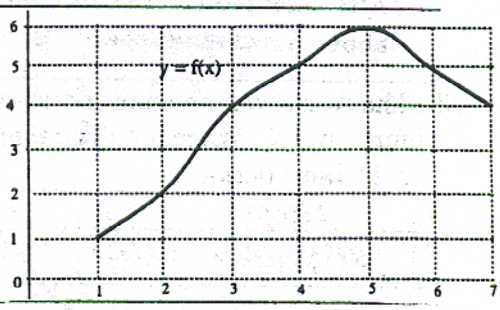
t	0	40	70	90	100
$g(t)$	150	180	195	184	172

(a) Estimate $\int_0^{150} g(t)\,dt$ by using a left Riemann sum with four subintervals.

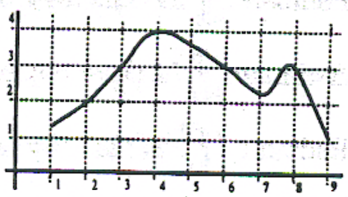
(b) Estimate $\int_0^{150} g(t)\,dt$ by using a right Riemann sum with four subintervals.

(c) Estimate $\int_0^{150} g(t)\,dt$ by using the trapezoidal rule with four subintervals.

3. The graph of the function f over the interval [1, 7] is shown. Using values from the graph, find trapezoidal rule estimates for the integral $\int_1^7 f(x)\,dx$ by using the indicated number of subintervals.
(a) n = 3
(b) n = 6



4. The graph of f over the interval [1, 9] is shown in the figure. Using the data in the figure, find a midpoint approximation with 4 equal subdivisions for $\int_1^9 f(x)\,dx$.



5. An experiment was performed in which oxygen was produced at a continuous rate. The rate at which oxygen was produced was measured each minute and the results tabulated.

minutes	0	1	2	3	4	5	6
oxygen (cu ft/min)	0	1.4	1.8	2.2	3.0	4.2	3.6

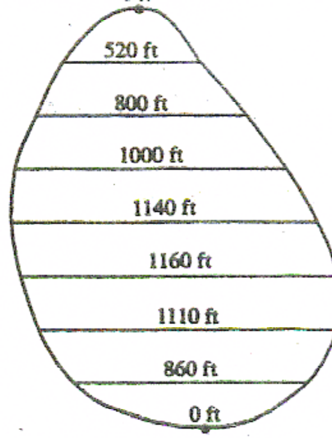
Use the trapezoid rule to estimate the total amount of oxygen produced in 6 minutes.

TURN--->>>

6. *Stocking a Fish Pond* As the fish and game warden of your township, you are responsible for stocking the town pond with fish before the fishing season. The average depth of the pond is 20 feet. Using a scaled map, you measure distances across the pond at 200-foot intervals, as shown in the diagram.

(a) Use the Trapezoidal Rule to estimate the volume of the pond.

(b) You plan to start the season with one fish per 1000 cubic feet. You intend to have at least 25% of the opening day's fish population left at the end of the season. What is the maximum number of licenses the town can sell if the average seasonal catch is 20 fish per license?



Vertical spacing = 200 ft

7. Bernhard Riemann V is planning to take a Sunday afternoon trip in his 1976 Mercedes Benz whose odometer broke when it turned over 200,000 miles. Being the descendant of the mathematician Georg Friedrich Bernhard Riemann (1826–1866), he decides to estimate the miles driven by recording his speed at various times during the trip. Because this is to be a restful trip and not a military drill, he does not plan to take readings at specified intervals of time, but only when he remembers to do so. He takes a stopwatch to record the time intervals (in minutes) to make the calculations easier for himself (and consequently for you). The following table contains the data he collects during his trip. Approximately how far does he travel? Each speedometer reading (in MPH) is made at some point during the time interval recorded.

time interval	15	25	30	15	20	35	40	20	10
speed	30	45	50	25	45	55	50	35	25

After completing the arithmetic to find the distance he has driven, Bernhard wonders about his average speed. Can you tell him what it is?

8. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table below shows the rate as measured every four hours for a 24-hour period.

t (hrs)	0	4	8	12	16	20	24
$R(t)$ (gal/hr)	7.5	9.0	9.3	9.5	8.8	8.0	7.2

(a) Estimate $\int_0^{24} R(t)\,dt$ by using a left Riemann sum with six subdivisions of equal length.

(b) Estimate $\int_0^{24} R(t)\,dt$ by using a right Riemann sum with six subdivisions of equal length.

(c) Estimate $\int_0^{24} R(t)\,dt$ by using a midpoint Riemann sum with three subdivisions of equal length.

(d) Estimate $\int_0^{24} R(t)\,dt$ by using the trapezoidal rule with six subdivisions of equal length.

(e) The rate of water flow, $R(t)$, can be approximated by the model, $W(t)=\frac{1}{75}\left(600+20t-t^2\right)$. Sketch the graph of $W(t)$ on the interval $0 \le t \le 24$.

(f) Use the model given in part (e) to approximate the number of gallons that have flowed out of the pipe during the 24-hour period.

(g) Use the model given in part (e) to approximate the average rate of water flow during the 24-hour period. At what time(s), $0 \le t \le 24$, does the water flow at its average rate?