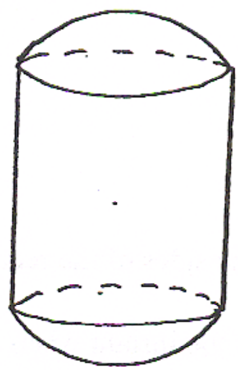
CALCULUS
WORKSHEET ON RELATED RATES

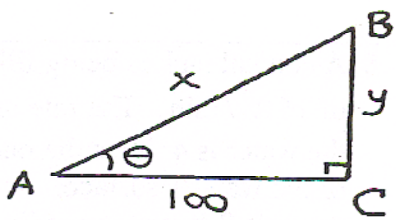
1. A paper cup, which is in the shape of a right circular cone, is 16 cm deep and has a radius of 4 cm. Water is poured into the cup at a constant rate of $2\,\text{cm}^3/\text{sec}$.
   (a) At the instant the depth is 5 cm, what is the rate of change of the height?
   (b) At the instant the radius is 3 cm, what is the rate of change of the radius?

2. A snowball is in the shape of a sphere. Its volume is increasing at a constant rate of $10\ \text{in}^3/\text{min}$.
   (a) How fast is the radius increasing when the volume is $36\pi\ \text{in}^3$?
   (b) How fast is the surface area increasing when the volume is $36\pi\ \text{in}^3$?

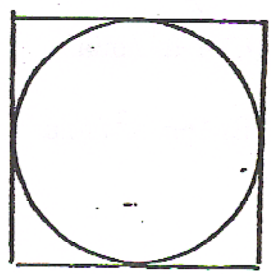
3. (1985) The balloon shown is in the shape of a cylinder with hemispherical ends of the same radius as that of the cylinder. The balloon is being inflated at the rate of $261\pi\,\text{cm}^3/\text{min}$. At the instant that the radius of the cylinder is 3 cm., the volume of the balloon is $144\pi\,\text{cm}^3$, and the radius of the cylinder is increasing at the rate of 2 cm/min.
   (a) At this instant, what is the height of the cylinder?
   (b) At this instant, how fast is the height of the cylinder increasing?



4. (1988) The figure shown represents an observer at point A watching balloon B as it rises from point C. The balloon is rising at a constant rate of 3 m/sec, and the observer is 100 m from point C.
   (a) Find the rate of change in x at the instant when $y = 50$.
   (b) Find the rate of change in the area of right triangle BCA at the instant when $y = 50$.
   (c) Find the rate of change of θ at the instant when $y = 50$.



5. (1994) A circle is inscribed in a square, as shown in the figure. The circumference of the circle is increasing at a constant rate of 6 in/sec. As the circle expands, the square expands to maintain the condition of tangency.
   (a) Find the rate at which the perimeter of the square is increasing.
   (b) At the instant when the area of the circle is $25\pi\ \text{in}^2$, find the rate of increase in the area enclosed between the circle and the square.



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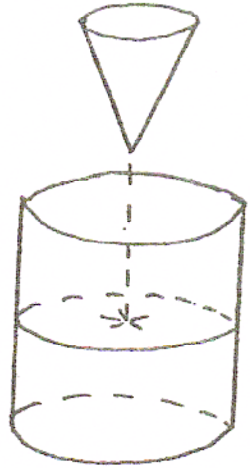
6. (1995) As shown in the figure, water is draining from a conical tank with height 12 ft and diameter 8 ft into a cylindrical tank that has a base with area 400π ft^2. The depth, h, in feet, of the water in the conical tank is changing at the rate of $(h-12)$ ft per minute.

(a) Write an expression for the volume of water in the conical tank as a function of h.

(b) At what rate is the volume of water in the conical tank changing when $h = 3$?

(c) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h = 3$?



7.

The sides of the rectangle above increase in such a way that $\frac{dz}{dt} = 1$ and $\frac{dx}{dt} = 3\frac{dy}{dt}$.

At the instant when $x = 4$ and $y = 3$, what is the value of $\frac{dx}{dt}$?

(A) $\frac{1}{3}$ (B) 1 (C) 2 (D) $\sqrt{5}$ (E) 5

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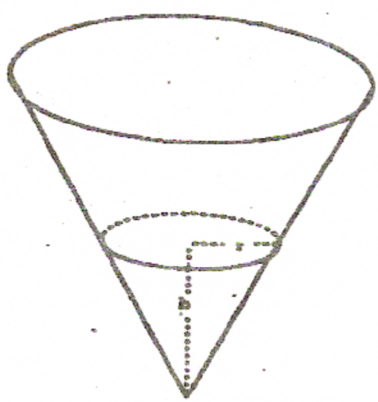
8. A conical tank is being filled with water at the rate of $16\,\text{ft}^3/\text{min}$. The rate of change of the depth of the water is 4 times the rate of change of the radius of the water's surface. At the moment when the depth is 8 ft. and the radius of the surface is 2 ft., the area of the surface is changing at the rate of



(A) $\frac{1}{\pi}$ ft^2/min (B) 1 ft^2/min

(C) 4 ft^2/min (D) 4π ft^2/min

(E) 16π ft^2/min