

CALCULUS  
 WORKSHEET ON DERIVATIVES (Second Deriv Test)

Work the following on notebook paper except for problems 11 – 12. Do not use your calculator. On problems 1 – 4, find the critical points of each function, and determine whether they are relative maximums or relative minimums by using the Second Derivative Test whenever possible.

1.  $f(x) = x^3 - 3x^2 + 3$
2.  $f(x) = x + \frac{4}{x}$
3.  $f(x) = \sin x - \cos x, 0 \leq x \leq 2\pi$
4.  $f(x) = 2\sin x + \cos(2x), 0 \leq x \leq 2\pi$

5. Suppose that the function  $f$  has a continuous second derivative for all  $x$  and that  $f(-1) = 2, f'(-1) = -3, f''(-1) = 5$ . Let  $g$  be a function whose derivative is given by  $g'(x) = (x^4 - 6x^3)(3f(x) + 2f'(x))$  for all  $x$ .

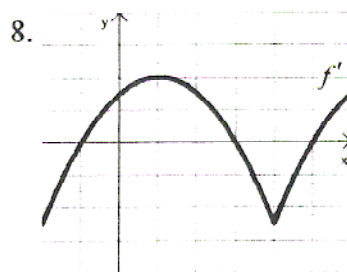
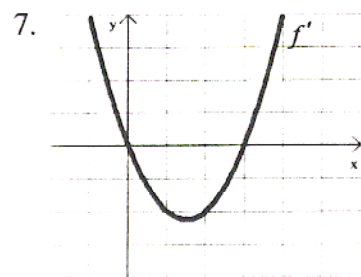
- (a) Write an equation of the line tangent to the graph of  $f$  at the point where  $x = -1$ .
- (b) Does  $g$  have a local maximum or a local minimum at  $x = -1$ ? Justify your answer.

6. Consider the curve given by  $x^2 + 4y^2 = 7 + 3xy$ .

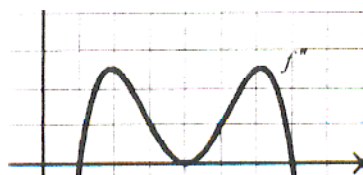
- (a) Show that  $\frac{dy}{dx} = \frac{3y - 2x}{8y - 3x}$ .
- (b) Show that there is a point  $P$  with  $x$ -coordinate 3 at which the line tangent to the curve at  $P$  is horizontal. Find the  $y$ -coordinate of  $P$ .
- (c) Find the value of  $\frac{d^2y}{dx^2}$  at the point  $P$  found in part (b). Does the curve have a local maximum, a local minimum, or neither at point  $P$ ? Justify your answer.

On problems 7 – 8, the graph of the derivative,  $f'$ , of a function  $f$  is shown.

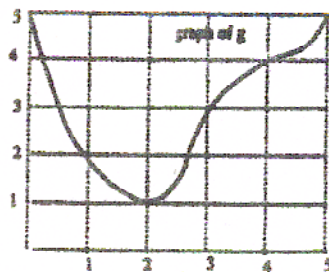
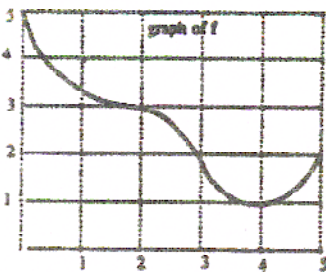
- (a) On what interval(s) is  $f$  increasing or decreasing? Justify your answer.
- (b) At what value(s) of  $x$  does  $f$  have a local maximum or local minimum? Justify your answer.



9. The graph of the second derivative,  $f''$ , of a function  $f$  is shown. State the  $x$ -coordinates of the inflection points of  $f$ . Justify your answer.



10. The function  $h(x) = f(g(x))$ , where  $f$  and  $g$  are functions whose graphs are shown below.



- Evaluate  $h(2)$ .
- Estimate  $h'(1)$ .
- Is the graph of the composite function  $h$  increasing or decreasing at  $x = 3$ ? Show your reasoning.
- Find all values of  $x$  for which the graph of  $h$  has a horizontal tangent. Show your reasoning.

11. For what values of  $a$  and  $b$  does the function  $f(x) = x^3 + ax^2 + bx + 2$  have a local maximum when  $x = -3$  and a local minimum when  $x = -1$ ?

12. Sketch the function  $f(x)$  from the following information:

- The domain of  $f$  is  $(-\infty, 0) \cup (0, \infty)$ .
- $f(-x) = -f(x)$
- $\lim_{x \rightarrow 0^+} f(x) = \infty$
- $\lim_{x \rightarrow \infty} f(x) = 0$
- For  $x > 0$ ,  $f(x) = 0$  only at  $x = 1$ .
- For  $x > 0$ ,  $f'(x) = 0$  only at  $x = 2$ .
- For  $x > 0$ ,  $f''(x) = 0$  only at  $x = 3$ .

